Optimal Effort in a Two-Period Model *

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This paper develops a generic model of effort in two periods wherein an individual exerts effort in monetary terms in the first period so as to improve risk in the second period. We specify such a risk improvement by means of a linear combination of two fixed probability distribution functions that can be ranked via first-order stochastic dominance. When the two fixed probability distribution functions degenerate into singletons, our two-period model of effort reduces to the two-period model of self-protection. Within an intertemporal framework with Kreps-Porteus-Selden preferences, we examine the comparative statics of effort with respect to the prevalence of uncertainty. We further examine the precautionary motive of saving in our two-period model of effort wherein the underlying uncertainty is endogenously determined by the choice of effort. Finally, we show that our results are consistent with recent experimental evidence for the negative relationship between prudence and self-protection regardless of the timing of loss.

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1. Introduction

Many economic and financial decision problems feature decision makers who take costly actions now so as to affect risky outcomes in the future to their favor. For example, firms that innovate incur R&D expenditures to improve the chance that their existing products can be upgraded and generate better risky free cash flows. Managers are motivated to work hard so as to increase their compensation that is tied to firm performance, albeit subject to various shocks that are out of their control. To delineate these intertemporal decision problems by a generic model, we extend the one-period model of effort *à la* Crainich et al. (2016) to a two-period setting.

Following Jindapon and Neilson (2007), Crainich et al. (2016) consider a decision maker who faces two risks that can be ranked via first-order stochastic dominance (FSD).[1] The decision maker exerts effort in monetary terms to reduce the chance of getting the less favorable risk and increase the chance of getting the more favorable risk. In the limiting case that the probability distribution functions of the two risks degenerate into singletons, the one-period effort model of Crainich et al. (2016) collapses into the celebrated self-protection model of Ehrlich and Becker (1972) wherein an individual spends on self-protection activities to reduce the probability of facing a fixed loss and increase the probability of facing no loss.[2]

In our two-period model of effort, an individual exerts effort in monetary terms in the first period so as to improve risk in the second period. As in Crainich et al. (2016), we specify such a risk improvement by means of a linear combination of two fixed probability distribution functions that can be ranked via FSD.[3] To disentangle the effect of risk preferences from that of time preferences on the individual's effort choice in an intertemporal setting, we

[1]Indeed, the model of Crainich et al. (2016) complements that of Jindapon and Neilson (2007) in that the latter uses the concept of more nth degree risk (Ekern, 1980) to order the two risks for all $n \geq 2$, whereas the former focuses on the case of FSD, i.e., $n = 1$.

[2]For other one-period models with general probability distribution functions, see Meyer and Meyer (2011) and Chiu (2012).

[3]One interpretation of such a linear specification is that the more favorable risk describes a target that the decision maker wants to achieve and the less favorable risk describes the status quo when the target is missed. More effort makes it more likely to reach the target (Chuang et al., 2013; Wong, 2017).

adopt the recursive utility model of Kreps and Porteus (1978) and Selden (1978) (hereafter referred to as Kreps-Porteus-Selden preferences), which has axiomatic foundations and includes the additive expected utility model as a special case. When the two fixed probability distribution functions degenerate into singletons and the additive expected utility model is used, our two-period model of effort reduces to the two-period self-protection model of Menegatti (2009).

We first examine the comparative statics of effort with respect to the prevalence of uncertainty, which is observationally equivalent to the comparison between the optimal effort level under risk aversion and that under risk neutrality. We show that the individual whose risk preferences exhibit prudence has a precautionary incentive to shift wealth from the first period to the second period by exerting more effort (Kimball, 1990, 1993).[4] This precautionary incentive is reinforced by the risk aversion incentive when the optimal effort level under certainty is sufficiently large such that the probability of getting the less favorable risk does not exceed a predetermined threshold. Indeed, these two incentives work in a similar fashion as in Menegatti (2009). However, our intertemporal framework with Kreps-Porteus-Selden preferences gives rise to an additional incentive due to consumption smoothing, which can induce the individual to adjust effort in either direction. Without restricting the individual's intertemporal preferences, we derive more stringent, yet intuitive, sufficient conditions under which the individual whose risk preferences exhibit prudence optimally exerts more effort when the uncertainty prevails. This calls for imposing a tighter upper bound on the probability of getting the less favorable risk at the optimal effort level under certainty so as to preclude the consumption smoothing incentive from overruling the other two incentives.

Masuda and Lee (2019) have recently conducted lab experiments to test the relationship between prudence and self-protection implied by the one-period model of Eeckhoudt and Gollier (2005) and that implied by the two-period model of Menegatti (2009).[5] Specifically,

[4]Prudence measures the propensity to prepare and forearm oneself under uncertainty, vis-à-vis risk aversion that is how much one dislikes uncertainty and would turn away from it if one could (Gollier, 2001).

[5]Krieger and Mayrhofer (2017) also conduct a lab experiment to study the relationship between prudence

subjects are assigned to one of two variants of the self-protection game, depending on the timing of the loss (current loss or future loss), wherein risk-neutral subjects optimally choose the level of self-protection at which the probability of loss is set equal to one-half. The results of Masuda and Lee (2019) are intriguing in that subjects spend less on self-protection than the risk-neutral level regardless of the timing of loss. These results are in line with those of Eeckhoudt and Gollier (2005), but contradictory to those of Menegatti (2009). We show that our results are perfectly consistent with the findings of Masuda and Lee (2019), thereby offering a plausible explanation based on Kreps-Porteus-Selden preferences.

In an intertemporal setting, it is natural to incorporate saving opportunities into our two-period model of effort. We are particularly interested in studying the precautionary motive of saving, i.e., the individual's motive to save at least the same amount in the presence than in the absence of uncertainty. We first show that saving dominates effort in smoothing consumption between the two periods, which leads to the immediate consequence that the optimal effort level is solely determined by equating the expected marginal benefit of effort to that of saving in the second period. The individual's time preferences as such become irrelevant to his effort choice, thereby reducing the two-period effort choice problem to a one-period effort choice problem once endogenous saving is allowed (Hofmann and Peter, 2016; Peter, 2017). We characterize prudence with Kreps-Porteus-Selden preferences, which is defined by means of precautionary saving, by either restricting risk preferences to satisfy non-increasing absolute risk aversion (Gollier, 2001; Kimball and Weil, 2009) or focusing on preferences for late risk resolution when risk preferences are quadratic. Since the underlying uncertainty is endogenously determined in our two-period model of effort via the choice of effort, we show that prudence with Kreps-Porteus-Selden preferences unambiguously leads to at least the same amount of saving when the uncertainty prevails if, and only if, the optimal effort level under uncertainty does not exceed that under certainty. As such, effort and saving in our two-period model exhibit a kind of substitutability at the optimum, even

and self-protection implied by a one-period model. They find that prudence is negatively associated with self-protection, confirming the theoretical findings of Eeckhoudt and Gollier (2005).

though the optimal schedule of saving, i.e., the optimal amount of saving as a function of effort, does not necessarily decrease when effort increases.

As pointed out by the literature on choice bracketing (Read et al., 1999; Thaler, 1999), when making multiple decisions, individuals can either broadly bracket them by assessing the consequences of all decisions taken together, or narrowly bracket them by making each decision in isolation. Since the determination of choice bracketing is not well understood, it is a priori unclear whether a given set of decisions is more susceptible to broad bracketing or to narrow bracketing. Furthermore, the same set of decisions might be bracketed broadly by one individual and narrowly by another. As such, we analyze our two-period model of effort with and without saving opportunities. Each scenario is relevant for our understanding of effort choices under uncertainty in its own right and thus deserves a close scrutiny.

Our two-period model of effort includes the two-period model of self-protection as a special case, thereby contributing to the large literature on self-protection initiated by the seminal work of Ehrlich and Becker (1972) (see, e.g., Briys and Schlesinger, 1990; Dionne and Eeckhoudt, 1985; Lee, 1998; Jullien et al., 1999; Dachraoui et al., 2004; Denuit, et al., 2016; Wong, 2016).[6] Crainich et al. (2016) consider a one-period model of effort similar to ours and their focus is on the effect of changes in either the more favorable risk or the less favorable risk on effort. In contrast, we follow Eeckhoudt and Gollier (2005) and Menegatti (2009) to examine how the prevalence of uncertainty affects the optimal effort choice. In this regard, our results are directly comparable to those of the extant literature.[7] Hofmann and Peter (2016) and Peter (2017) develop additive expected utility models of self-protection and saving in two periods and show that their intertemporal models of self-protection collapse into static one-period models once endogenous saving is allowed. We extend this finding to the case of recursive utility with Kreps-Porteus-Selden preferences and examine the precautionary motive of saving in this more general context. Wang and

[6] Indeed, when there are saving opportunities, our two-period model of effort collapses into a one-period model so that the one-period model of self-protection is also a special case of ours.

[7] In a recent article, Wang et al. (2019) examine the effect of greater risk and risk aversion on effort with Kreps-Porteus-Selden preferences, which is in line with the analysis of Crainich et al. (2016).

Li (2016) and Bostian and Heinzel (2018) also study the intensity of precautionary saving with Kreps-Porteus-Selden preferences. Unlike them, we allow the underlying uncertainty to be endogenously determined by the choice of effort, thereby bringing new insights into the precautionary motive of saving.

The rest of the paper is organized as follows. Section 2 develops a generic model of effort in two periods. Section 3 examines the comparative statics of effort with respect to the prevalence of uncertainty. Section 4 incorporates saving opportunities into our two-period model of effort and examines the precautionary motive of saving. The final section concludes.

2. The model

Consider an individual who lives for two periods and is endowed with a known wealth level, $w_\circ > 0$, in the first period and an uncertain wealth level, $\tilde{w} > 0$, in the second period.[8] The individual can exert effort, $e \in [0, w_\circ]$, ex ante so as to improve the cumulative distribution function (CDF) of the uncertain second-period wealth, $\tilde{w}$, where effort is measured in monetary terms. We specify the CDF of $\tilde{w}$ as follows:

$$H(w|e) = p(e)F(w) + [1 - p(e)]G(w), \tag{1}$$

over support $[a, b]$, where $0 < a < b$, $p(e) \in (0, 1)$, and $F(w)$ and $G(w)$ are two fixed CDFs of $\tilde{w}$ over support $[a, b]$ such that $G(w)$ dominates $F(w)$ via first-order stochastic dominance (FSD), i.e., $F(w) \geq G(w)$ for all $w \in [a, b]$.[9] We assume that $p'(e) < 0$ for all $e \in [0, w_\circ]$ so that $H(w|e_2)$ dominates $H(w|e_1)$ via FSD whenever $e_2 > e_1$, as is evident from Eq. (1). As such, the individual's first-period consumption, $c_1 = w_\circ - e$, is certain, whereas his

[8] Throughout the paper, random variables have a tilde (~) while their realizations do not.

[9] The specification of $H(w|e)$ in Eq. (1) is adopted from Jindapon and Neilson (2007) with one caveat. While we focus on the case that $G(w)$ dominates $F(w)$ via FSD, Jindapon and Neilson (2007) consider the case that $F(w)$ has more nth-degree risk than $G(w)$ in the sense of Ekern (1980) for all $n \geq 2$. It is worth pointing out that the results of Jinadpon and Neilson (2007) are not applicable when $G(w)$ dominates $F(w)$ via FSD.

second-period consumption, $\tilde{c}_2 = \tilde{w}$, is uncertain and distributed according to $H(w|e)$.

Denote $\mathrm{E}_F[\cdot]$, $\mathrm{E}_G[\cdot]$, and $\mathrm{E}_H[\cdot]$ as the expectation operators with respect to $F(w)$, $G(w)$, and $H(w|e)$, respectively. Let $\mu_F > 0$ and $\sigma_F^2 \geq 0$ be the expected value and variance of $\tilde{w}$ with respect to $F(w)$. Likewise, let $\mu_G > 0$ and $\sigma_G^2 \geq 0$ be the expected value and variance of $\tilde{w}$ with respect to $G(w)$. Since $G(w)$ dominates $F(w)$ via FSD, it must be true that $\mu_G - \mu_F = \ell > 0$. However, σ_F^2 can be smaller than, equal to, or greater than σ_G^2. Using Eq. (1), the expected value and variance of $\tilde{w}$ with respect to $H(w|e)$ is $\mu_H(e) = p(e)\mu_F + [1 - p(e)]\mu_G = \mu_G - p(e)\ell$ and

$$\sigma_H^2(e) = \int_a^b [w - \mu_H(e)]^2 \mathrm{d}H(w|e) = p(e)\sigma_F^2 + [1 - p(e)]\sigma_G^2 + p(e)[1 - p(e)]\ell^2, \tag{2}$$

respectively. Since $\mu_H'(e) = -p'(e)\ell > 0$ for all $e \in [0, w_\circ]$, $\mu_H(e)$ is always increasing in e. Differentiating Eq. (2) with respect to e yields

$$\sigma_H^{2\prime}(e) = 2p'(e)\ell^2\left[\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2} - p(e)\right]. \tag{3}$$

It follows from Eq. (3) that $\sigma_H^2(e)$ decreases with an increase (a decrease) in effort for all e such that $p(e) < (>)\ 1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2$, and is invariant to a marginal change in effort at the unique point, e, that solves $p(e) = 1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2$.

Our two-period model of effort is adapted from that of Peter (2017), which in turn is an extension of the one-period model of Crainich et al. (2016). A special case that is of great interest in the literature is the one wherein $\tilde{w} \equiv \mu_F$ under $F(w)$ and $\tilde{w} \equiv \mu_G$ under $G(w)$ so that $G(w)$ trivially dominates $F(w)$ via FSD given that $\mu_F < \mu_G$ and $\sigma_F^2 = \sigma_G^2 = 0$. In this special case, we can interpret μ_G as the certain second-period wealth that is subject to a binary risk of facing a fixed loss, $\ell = \mu_G - \mu_F > 0$, with probability $p(e)$ and no loss with probability $1 - p(e)$. This is precisely the self-protection model of Ehrlich and Becker (1972) wherein e is the expenditure on self-protection activities that can reduce the probability of loss, $p(e)$.[10]

[10] Courbage and Rey (2012), Eeckhoudt et al. (2012), and Wang and Li (2015) consider another important special case wherein $F(w)$ is a leftward parallel shift of $G(w)$ by a deterministic additive term.

Owing to the intertemporal nature of the two-period setting, individual u's effort choice depends not only on his risk preferences but also on his time preferences. In order to separate the effect of risk preferences from that of time preferences on effort, we follow Kimball and Weil (2009) to use recursive utility *à la* Kreps and Porteus (1978) and Selden (1978) (hereafter referred to as Kreps-Porteus-Selden preferences). Specifically, the individual has preferences over certain first-period and uncertain second-period consumption pairs, $(c_1, \tilde{c}_2)$, which are described by the following additive recursive utility (Gollier, 2001; Kimball and Weil, 2009; Wang and Li, 2016; Bostian and Heinzel, 2018):

$$\Phi(c_1, \tilde{c}_2) = u_1(c_1) + u_2\Big(u^{-1}\Big(\mathrm{E}_H[u(\tilde{c}_2)]\Big)\Big), \tag{4}$$

where $u_1(c_1)$ and $u_2(c_2)$ are two felicity functions capturing the time preferences over certain (c_1, c_2)-pairs, and $u(c_2)$ is a von Neumann-Morgenstern utility function that represents the risk preferences over $\tilde{c}_2$.[11] The individual is risk averse so that $u'(c_2) > 0$ and $u''(c_2) < 0$ for all $c_2 \in [a, b]$. The pair of felicity functions, $\big(u_1(c_1), u_2(c_2)\big)$, is referred to as the time aggregator. We assume that $u_1(c_1)$ is concave, i.e., $u_1'(c_1) > 0$ and $u_1''(c_1) < 0$ for all $c_1 \in [0, w_\circ]$, and $u_2(c_2)$ is weakly concave, i.e., $u_2'(c_2) > 0$ and $u_2''(c_2) \leq 0$ for all $c_2 \in [a, b]$.

The representation in Eq. (4) reveals that the individual's lifetime utility is characterized in two steps. First, the certainty equivalent of the uncertain second-period consumption, $\tilde{c}_2$, is computed using $u(c_2)$, which gives rise to $u^{-1}\big(\mathrm{E}_H[u(\tilde{c}_2)]\big)$. Since the first step is conducted in an atemporal context, the concavity of $u(c_2)$ measures the degree of risk aversion alone. Second, the lifetime utility is evaluated by adding the utility of the first-period consumption to that of the certainty equivalent of the uncertain second-period consumption by means of the time aggregator, $\big(u_1(c_1), u_2(c_2)\big)$. Since the second step is conducted after all uncertainty has been eliminated, the concavity of $u_1(c_1)$ and the weak concavity of $u_2(c_2)$ are related to preferences for consumption smoothing over time alone. As such, the individual's time and risk preferences are fully separable by the time aggregator, $\big(u_1(c_1), u_2(c_2)\big)$, and the von Neumann-Morgenstern utility function, $u(c_2)$, respectively.

[11] See Kreps and Porteus (1978) and Selden (1978) for the axiomatic foundations of $\Phi(c_1, \tilde{c}_2)$.

The function, $\phi(x) = u_2\Big(u^{-1}(x)\Big)$ for all $x \in [u(a), u(b)]$, is referred to as the Kreps-Porteus operator. The curvature of $\phi(x)$ reflects the individual's preferences for the timing of risk resolution. As shown by Kreps and Porteus (1978) (see also Gollier, 2001), the individual has preferences for early (late) risk resolution if the certainty equivalent of the uncertain second-period consumption, $\tilde{c}_2$, is always larger when it is computed using $u_2(c_2)$ than that using $u(c_2)$, which is true if, and only if, $u_2(c_2)$ is less (more) concave than $u(c_2)$ so that

$$\phi''(x) = \frac{u_2'\Big(u^{-1}(x)\Big)}{u'\Big(u^{-1}(x)\Big)^2}\left[\frac{u_2''\Big(u^{-1}(x)\Big)}{u_2'\Big(u^{-1}(x)\Big)} - \frac{u''\Big(u^{-1}(x)\Big)}{u'\Big(u^{-1}(x)\Big)}\right] > (<)\ 0, \tag{5}$$

for all $x \in [u(a), u(b)]$. When the Kreps-Porteus operator is linear, we have $u_2(c_2) = u(c_2)$ for all $c_2 \in [a, b]$ so that the Kreps-Porteus-Selden preferences collapse into the standard additive expected utility model that dose not allow for the separation of risk preferences from time preferences.

Holding the first-period consumption fixed at c_1, the elasticity of intertemporal substitution is given by

$$\eta(c_1, c_2) = -\frac{\mathrm{d}\ln(c_2/c_1)}{\mathrm{d}\ln[u_2'(c_2)/u_1'(c_1)]} = -\frac{u_2'(c_2)}{c_2 u_2''(c_2)}. \tag{6}$$

The reciprocal of $\eta(c_1, c_2)$ measures the resistance to intertemporal substitution (Gollier, 2001; Kimball and Weil, 2009). Using Eqs. (5) and (6), $\phi(x)$ is convex (concave) if, and only if, the resistance to intertemporal substitution, $1/\eta(c_1, c_2)$, is smaller (greater) than the measure of relative risk aversion, $-c_2 u''(c_2)/u'(c_2)$. The empirical evidence that individuals are more resistant to intertemporal substitution than they are risk averse is mixed (see Gollier, 2001), even though Hall (1988) once argues that the elasticity of intertemporal substitution is close to zero. Hence, the cases that $\phi(x)$ is convex, linear, and concave are all likely.

The individual's lifetime utility as a function of the effort level, e, is given by

$$U(e) = u_1(w_\circ - e) + u_2\Big(m(e)\Big), \tag{7}$$

where $m(e)$ is the certainty equivalent of $\tilde{c}_2$ with respect to $u(c_2)$. Specifically, $m(e)$ is implicitly defined by

$$u\Big(m(e)\Big) = p(e)\mathrm{E}_F[u(\tilde{w})] + [1 - p(e)]\mathrm{E}_G[u(\tilde{w})]. \tag{8}$$

The individual chooses an effort level, e, in the first period so as to maximize his lifetime utility defined in Eq. (7), which gives rise to the following first-order condition:

$$u_1'(w_\circ - e^*) = -\frac{u_2'\Big(m(e^*)\Big)}{u'\Big(m(e^*)\Big)} p'(e^*)\{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})]\}, \tag{9}$$

where e^* is the optimal effort level, and we have used Eq. (8). We assume that $U''(e) < 0$ for all $e \in [0, w_\circ]$ so that Eq. (9) is both necessary and sufficient for e^* to be the unique maximum solution.[12]

The left-hand side of Eq. (9) is the marginal cost of effort, which is measured by the reduction in first-period utility. The right-hand side of Eq. (9) is the expected marginal benefit of effort, which is measured by the increase in the expected second-period utility after controlling for the effect of time preferences by the adjustment factor, $u_2'(c_2)/u'(c_2)$, evaluated at $c_2 = m(e^*)$. Hence, Eq. (9) is the usual optimality condition under which the marginal cost of effort is equated to the expected marginal benefit of effort at the optimum. The adjustment factor, $u_2'(c_2)/u'(c_2)$, is related to the Kreps-Porteus operator, $\phi(x)$, in that $u_2'(c_2)/u'(c_2)$ is decreasing (increasing) in c_2 if, and only if, $\phi(x)$ is concave (convex), as is evident from Eq. (5).

3. Optimal effort

In this section, we examine the effect of uncertainty on effort. To this end, we consider the benchmark case of certainty in which the second-period wealth is certain and equal to

[12] Sufficient conditions that ensure $U(e)$ to be concave for all $e \in [0, w_\circ]$ are that $p''(e) \geq 0$ for all $e \in [0, w_\circ]$ and $u_2'(c_2)/u'(c_2)$ is non-increasing for all $c_2 \in [a, b]$.

$\mu_G - p(e)\ell$, where $\ell = \mu_G - \mu_F > 0$. The optimal effort level under certainty, e°, as such is the solution to the following first-order condition:[13]

$$u_1'(w_\circ - e^\circ) = -u_2'\Big(\mu_G - p(e^\circ)\ell\Big)p'(e^\circ)\ell. \tag{10}$$

It is worth pointing out that the benchmark case of certainty is observationally equivalent to the case of risk neutrality wherein $u(c_2) = c_2$. The risk-neutral optimal effort level as such is also e° that solves Eq. (10).

To compare the optimal effort level under uncertainty, e^*, with that under certainty, e°, we differentiate Eq. (7) with respect to e and evaluate the resulting derivative at $e = e^\circ$ to yield

$$U'(e^\circ) = -u_1(w_\circ - e^\circ) - \frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)}p'(e^\circ)\{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})]\}$$

$$= u_2'\Big(\mu_G - p(e^\circ)\ell\Big)p'(e^\circ)\ell - \frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)}p'(e^\circ)\{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})]\}, \tag{11}$$

where the second equality follows from Eq. (10). Given that $U''(e) < 0$ for all $e \in [0, w_\circ]$, Eq. (9) implies that $e^* > (<)\ e^\circ$ if, and only if, $U'(e^\circ) > (<)\ 0$. We derive sufficient conditions under which $e^* > (<)\ e^\circ$ in our first proposition.

Proposition 1. *Given that the individual has Kreps-Porteus-Selden preferences wherein the Kreps-Porteus operator is weakly concave (convex) and risk preferences exhibit prudence (imprudence) in that $u'''(c_2) > (<)\ 0$ for all $c_2 \in [a, b]$, the individual optimally exerts more (less) effort in the presence than in the absence of uncertainty, i.e., $e^* > (<)\ e^\circ$, if the optimal effort level under certainty, e°, satisfies that*

$$p(e^\circ) \leq (\geq)\ \frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}. \tag{12}$$

[13] A sufficient condition that ensures e° to be the unique maximum solution is that $p''(e) \geq 0$ for all $e \in [0, w_\circ]$.

Proof. See Appendix A. □

The intuition for Proposition 1 is as follows. Since the second-period wealth is uncertain, the individual whose risk preferences exhibit prudence (imprudence) in that $u'(c_2)$ is convex (concave), has a precautionary incentive to shift wealth from the first (second) period to the second (first) period by exerting more (less) effort (Kimball, 1990, 1993).[14] This precautionary incentive is interacted with other incentives arising from risk aversion and consumption smoothing. It follows from Eq. (3) that $\sigma_H^2(e)$ does not increase with an increase (a decrease) in effort from e° should e° satisfy condition (12). The risk aversion incentive as such does not counteract the precautionary incentive. Indeed, in the case that the Kreps-Porteus operator, $\phi(x)$, is linear, we have shown in the proof of Proposition 1 that prudence (imprudence) and condition (12) ensure that

$$-p'(e^\circ)\{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})]\} > (<) \; -u_2'\Big(\mu_G - p(e^\circ)\ell\Big)p'(e^\circ)\ell, \tag{13}$$

i.e., the prevalence of uncertainty raises (lowers) the expected marginal benefit of effort, thereby making the individual exert more (less) effort than e°.

Suppose that $u(c_2)$ is quadratic and $p(e^\circ) = 1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2$ so that $m(e^\circ) < \mu_G - p(e^\circ)\ell = \mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell$. In this case, both the precautionary and risk aversion incentives are made void so that inequality (13) holds as an equality. Eq. (11) becomes[15]

$$\begin{aligned} U'(e^\circ) = {} & -u'\left(\mu_G - \frac{\ell}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell}\right)p'(e^\circ)\ell \\ & \times \left[\frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)} - \frac{u_2'\Big(\mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell\Big)}{u'\Big(\mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell\Big)}\right]. \end{aligned} \tag{14}$$

When the Kreps-Porteus operator, $\phi(x)$, is linear, the right-hand side of Eq. (14) vanishes so that e° is indeed optimal for achieving consumption smoothing between the two periods.

[14] The expected second-period consumption is $\mu_H(e) = \mu_G - p(e)\ell$, which increases with an increase in effort.

[15] Since $u(c_2)$ is quadratic, we have $\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})] = u'\Big(\mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell\Big)\ell$.

On the other hand, when $\phi(x)$ is concave (convex), the adjustment factor, $u_2'(c_2)/u'(c_2)$, is decreasing (increasing) for all $c_2 \in [a, b]$ so that the right-hand side of Eq. (14) is positive (negative). The concavity (convexity) of $\phi(x)$ implies that the individual has preferences for late (early) risk resolution, which strengthen (weaken) the precautionary incentive, thereby inducing the individual to exert more (less) effort than e°.[16] Combining the consequences of the three incentives yields the results of Proposition 1.

When the Kreps-Porteus operator, $\phi(x)$, is linear, our model reduces to the additive expected utility model. As such, the results of Proposition 1 extend those of Menegatti (2009) in the setting of self-protection in that we allow the outcome in the loss state and that in the no-loss state to be uncertain and ranked by FSD. Unlike Menegatti (2009) who introduces a specific assumption on $w_\circ$ such that the first-period consumption is set equal to the expected second-period consumption at $e = e^\circ$, we employ Kreps-Porteus-Selden preferences to separate time and risk preferences. In the special case wherein the Kreps-Porteus operator, $\phi(x)$, is linear so that $u_2(c_2) = u(c_2)$ for all $c_2 \in [a, b]$, Eq. (10) becomes

$$u_1'(w_\circ - e^\circ) = -u'\Big(\mu_G - p(e^\circ)\ell\Big)p'(e^\circ)\ell. \tag{15}$$

Menegatti (2009) focuses on the case that $u_1(c_2) = u(c_2)$ and $-p'(e^\circ)\ell = 1$. Hence, in this case, Eq. (15) holds if, and only if, $w_\circ - e^\circ = \mu_G - p(e^\circ)\ell$, which is the assumption made by Menegatti (2009).

Independent of the Kreps-Porteus operator, $\phi(x)$, the consumption smoothing incentive is likely to counteract the precautionary and risk aversion incentives. The following proposition shows that $p(e^\circ)$ has to be bounded from above by a threshold that is strictly below $1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2$ to ensure that the risk aversion incentive is strong enough to preclude the consumption smoothing incentive from overruling the other two incentives. As such, the individual whose risk preferences exhibit prudence in that $u'(c_2)$ is convex unambiguously

[16] Proposition 4 shows that the weak concavity of $\phi(x)$ alone is both necessary and sufficient for precautionary saving when risk preferences are quadratic.

exerts more effort than e°.

Proposition 2. *Given that the individual has Kreps-Porteus-Selden preferences wherein risk preferences exhibit prudence in that $u'''(c_2) > 0$ for all $c_2 \in [a,b]$, and that the risk premium, π_G, which solves $u(\mu_G - \pi_G) = \mathrm{E}_G[u(\tilde{w})]$, is sufficiently small such that $\pi_G < \ell/2 + (\sigma_F^2 - \sigma_G^2)/2\ell$, the individual optimally exerts more effort in the presence than in the absence of uncertainty, i.e., $e^* > e^\circ$, if the optimal effort level under certainty, e°, satisfies that*

$$p(e^\circ) \leq \frac{\mathrm{E}_G[u(\tilde{w})] - u\Big(\mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell\Big)}{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})]} \in \left(0, \frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\right). \tag{16}$$

Proof. See Appendix B. □

The condition, $\pi_G < \ell/2 + (\sigma_F^2 - \sigma_G^2)/2\ell$, ensures that the threshold in condition (16) is positive. This condition is satisfied trivially in the model of self-protection, which implies that $\pi_G = 0$. Indeed, this condition holds as long as μ_F and μ_G differ by a big enough margin or σ_G^2 is sufficiently small.

Masuda and Lee (2019) design a lab experiment of the two-period model of self-protection by setting the risk-neutral optimal level of self-protection to satisfy that $p(e^\circ) = 1/2$.[17] Masuda and Lee (2019) find that risk-averse and prudent subjects choose their optimal levels of self-protection less than e°, which is inconsistent with the theoretical results of Menegatti (2009) but perfectly consistent with ours. While Proposition 2 says that risk-averse and prudent individuals have their optimal levels of self-protection exceeding e° if $p(e^\circ)$ is sufficiently below 1/2, it is possible that they may optimally choose to spend less than e° on self-protection when $p(e^\circ) = 1/2$. To see this, consider an example wherein $u_2(c_2) = c_2$ and $u(c_2) = \alpha c_2 - \beta c_2^2 + \gamma c_2^3$, where α, β, and γ are positive constants such that $\alpha - 2\beta c_2 + 3\gamma c_2^2 > 0$ and $-2\beta + 6\gamma c_2 < 0$. In this example, the Kreps-Porteus operator, $\phi(x)$, is convex and the

[17] Masuda and Lee (2019) use the functional form: $p(e) = 1/(1+ke)$ and $\ell = 4/k$, where k is a positive constant. The risk-neutral level of self-protection as such is $e^\circ = 1/k$ so that $p(e^\circ) = 1/2$.

utility function, $u(c_2)$, exhibits prudence so that the results of Proposition 1 do not apply. Setting $p(e^\circ) = 1/2$, we notice that Eq. (11) in the model of self-protection becomes

$$U'(e^\circ) = \frac{p'(e^\circ)\ell}{u'\Big(m(e^\circ)\Big)}\bigg\{\Big[\mu_G - \frac{\ell}{2} - m(e^\circ)\Big] \times\bigg\{2\beta - 6\gamma\Big[\frac{1}{2}m(e^\circ) + \frac{1}{2}\Big(\mu_G - \frac{\ell}{2}\Big)\Big]\bigg\} - \frac{\gamma\ell^3}{4}\bigg\}. \quad (17)$$

Given that $m(e^\circ) < \mu_G - \ell/2$ and $2\beta - 6\gamma c_2 > 0$, the right-hand side of Eq. (17) must be negative for γ sufficiently small. The risk-averse and prudent individual as such optimally spends less than e° on self-protection. In this example, the risk aversion incentive is muted because of $p(e^\circ) = 1/2$ and $\sigma_F^2 = \sigma_G^2 = 0$, which, from Eq. (3), ensure that $\sigma_H^2(e)$ is invariant to a marginal change in effort from e°. The convexity of the Kreps-Porteus operator implies that the individual has preferences for early risk resolution, which weaken the precautionary incentive. For γ sufficiently small, the precautionary incentive is already very weak so that the individual is ultimately induced to exert less effort than e°. We as such offer a plausible explanation for the findings of Masuda and Lee (2019) in the context of Kreps-Porteus-Selden preferences.[18]

4. Optimal effort and precautionary saving

In an intertemporal setting, it is natural to incorporate saving opportunities into our model. To this end, we allow the individual to save an endogenously chosen amount, $s \in [0, w_\circ - e]$, in the first period so as to receive Rs in the second period, where $R \geq 1$ is a known gross rate of interest. As such, the individual's first-period consumption, $c_1 = w_\circ - e - s$, is certain, whereas his second-period consumption, $\tilde{c}_2 = \tilde{w} + Rs$, is uncertain, where $\tilde{w}$ is distributed according to $H(w|e)$.

[18] Masuda and Lee (2019) provide an alternative explanation based on the concept of dual prudence using probability weighting (Eeckhoudt et al., 2017).

The individual's lifetime utility as a function of the effort and saving pair, (e, s), is given by

$$U(e,s) = u_1(w_\circ - e - s) + u_2\Big(m(e,s)\Big), \tag{18}$$

where $m(e, s)$ is implicitly defined by

$$u\Big(m(e,s)\Big) = p(e)\mathrm{E}_F[u(\tilde{w} + Rs)] + [1 - p(e)]\mathrm{E}_G[u(\tilde{w} + Rs)]. \tag{19}$$

The individual chooses a pair of effort and saving, (e, s), in the first period so as to maximize his lifetime utility defined in Eq. (18). To facilitate the exposition, we formulate the individual's decision problem as a two-stage optimization problem. In the first stage, we fix the effort level at e and solve the individual's optimal amount of saving, $s(e)$, as a function of e, which is the solution to the following first-order condition:[19]

$$U_s\Big(e, s(e)\Big) = -u_1'\Big(w_\circ - e - s(e)\Big) + \frac{u_2'\Big(m\big(e, s(e)\big)\Big)}{u'\Big(m\big(e, s(e)\big)\Big)} \times \Big\{p(e)\mathrm{E}_F\Big[u'\Big(\tilde{w} + Rs(e)\Big)\Big] + [1 - p(e)]\mathrm{E}_G\Big[u'\Big(\tilde{w} + Rs(e)\Big)\Big]\Big\}R = 0, \tag{20}$$

where $U_s(e, s) = \partial U(e, s)/\partial s$. Rearranging terms of Eq. (20) yields

$$\frac{u_1'\Big(w_\circ - e - s(e)\Big)}{\frac{u_2'\left(m\left(e,s(e)\right)\right)}{u'\left(m\left(e,s(e)\right)\right)}\Big\{p(e)\mathrm{E}_F\Big[u'\Big(\tilde{w} + Rs(e)\Big)\Big] + [1 - p(e)]\mathrm{E}_G\Big[u'\Big(\tilde{w} + Rs(e)\Big)\Big]\Big\}} = R. \tag{21}$$

The left-hand of Eq. (21) is the marginal rate of substitution of first-period consumption for second-period consumption, which is equated to the relative price of first-period consumption in terms of second-period consumption, R, at the optimum. Hence, consumption smoothing is achieved for all e when the individual saves the amount, $s(e)$.

[19] The sign of $s'(e)$ is indeterminate. A sufficient condition for $s'(e) < 0$ is that the Kreps-Porteus operator, $\phi(x)$, is weakly concave. A necessary condition for $s'(e) > 0$ is that $\phi(x)$ is convex.

In the second stage, taking the schedule of saving, $s(e)$, as given, we solve the individual's optimal effort level, e^*, which is the solution to the following first-order condition:

$$U_e\Big(e^*, s(e^*)\Big) = -u_1'\Big(w_\circ - e^* - s(e^*)\Big) - \frac{u_2'\Big(m\Big(e^*, s(e^*)\Big)\Big)}{u'\Big(m\Big(e^*, s(e^*)\Big)\Big)}$$

$$\times p'(e^*)\Big\{\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^*)\Big)\Big] - \mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^*)\Big)\Big]\Big\} = 0, \tag{22}$$

where $U_e(e, s) = \partial U(e, s)/\partial e$ and we have used Eq. (20) with $e = e^*$. Substituting Eq. (20) with $e = e^*$ into Eq. (22) and rearranging terms yields

$$-p'(e^*)\left\{\frac{\mathrm{E}_G[u(\tilde{w} + Rs^*)] - \mathrm{E}_F[u(\tilde{w} + Rs^*)]}{p(e^*)\mathrm{E}_F[u'(\tilde{w} + Rs^*)] + [1 - p(e^*)]\mathrm{E}_G[u'(\tilde{w} + Rs^*)]}\right\} = R, \tag{23}$$

where $s^* = s(e^*)$. The term, $\mathrm{E}_G[u(\tilde{w} + Rs^*)] - \mathrm{E}_F[u(\tilde{w} + Rs^*)]$, is the utility premium that measures the "pain" associated with facing the passage from the more favorable CDF of $\tilde{w}$, $G(w)$, to the less favorable CDF of $\tilde{w}$, $F(w)$ (Eeckhoudt and Schlesinger, 2006). This utility premium is normalized by the expected marginal utility at the optimum, $p(e^*)\mathrm{E}_F[u'(\tilde{w} + Rs^*)] + [1 - p(e^*)]\mathrm{E}_G[u'(\tilde{w} + Rs^*)]$, so that the normalized utility premium is invariant to any positive affine transformations of $u(c_2)$. Eq. (23) simply states that the expected marginal benefit of effort based on the normalized utility premium is equated to the gross rate of interest at the optimum, where the latter measures the forgone second-period consumption should the effort level be increased by a marginal unit. Since Eq. (23) holds in the second period, the individual's time preferences no longer matter for his effort choice. The two-period effort choice problem as such reduces to a one-period effort choice problem once endogenous saving is allowed (Hofmann and Peter, 2016; Peter, 2017).

We assume that $U(e, s)$ is concave for all $(e, s) \in [0, w_\circ] \times [0, w_\circ]$ so that Eqs. (20) and (22) are both necessary and sufficient for (e^*, s^*) to be the unique maximum solution.[20]

[20]See Appendix E for the sufficient conditions that ensure $U(e, s)$ to be concave for all $(e, s) \in [0, w_\circ] \times [0, w_\circ]$.

As a benchmark, we consider the case wherein the second-period wealth is certain and equal to $\mu_G - p(e)\ell$, where $\ell = \mu_G - \mu_F > 0$. In this certainty case, the individual's schedule of saving, $\hat{s}(e)$, is the solution to the following first-order condition:

$$u_1'\Big(w_\circ - e - \hat{s}(e)\Big) = u_2'\Big(\mu_G - p(e)\ell + R\hat{s}(e)\Big)R. \tag{24}$$

Differentiating Eq. (24) with respect to e and rearranging terms yields

$$\hat{s}'(e) = \frac{-u_1''\Big(w_\circ - e - \hat{s}(e)\Big) + u_2''\Big(\mu_G - p(e)\ell + R\hat{s}(e)\Big)Rp'(e)\ell}{u_1''\Big(w_\circ - e - \hat{s}(e)\Big) + u_2''\Big(\mu_G - p(e)\ell + R\hat{s}(e)\Big)R^2} < 0, \tag{25}$$

so that effort and saving are substitutes. Taking the schedule of saving, $\hat{s}(e)$, as given, the individual's optimal effort level under certainty, e°, is the solution to the following first-order condition:

$$u_1'\Big(w_\circ - e^\circ - \hat{s}(e^\circ)\Big) = -u_2'\Big(\mu_G - p(e^\circ)\ell + R\hat{s}(e^\circ)\Big)p'(e^\circ)\ell. \tag{26}$$

Substituting Eq. (24) with $e = e^\circ$ and $\hat{s}(e^\circ) = s^\circ$ into Eq. (26) yields

$$-p'(e^\circ)\ell = R. \tag{27}$$

It then follows from Eq. (25) with $e = e^\circ$ and Eq. (27) that $\hat{s}'(e^\circ) = -1$, i.e., effort and saving are perfect substitutes at the optimum under certainty. As such, the individual's optimal effort level under certainty, e°, is the one that maximizes the certain second-period consumption given by $\mu_G - p(e)\ell + R\hat{s}(e)$.

4.1 Prudence and precautionary saving

Following Kimball (1990) and Gollier (2001), we define prudence with Kreps-Porteus-Selden preferences by means of precautionary saving. Specifically, we say that the individual with Kreps-Porteus-Selden preferences is prudent independent of the Kreps-Porteus operator,

$\phi(x)$, if he optimally saves at least the same amount in the presence than in the absence of uncertainty, i.e., $s(e) \geq \hat{s}(e)$, for any fixed effort level, e, for any two CDFs of $\tilde{w}$, $F(w)$ and $G(w)$, such that $G(w)$ dominates $F(w)$ via FSD, and any time aggregator, $\Big(u_1(c_1), u_2(c_2)\Big)$, such that $u_1(c_1)$ is concave and $u_2(c_2)$ is weakly concave.

To compare $s(e)$ with $\hat{s}(e)$, we differentiate Eq. (18) with respect to s and evaluate the resulting derivative at $s = \hat{s}(e)$ to yield

$$
\begin{aligned}
U_s\Big(e, \hat{s}(e)\Big) &= -u_1'\Big(w_\circ - e - \hat{s}(e)\Big) + \frac{u_2'\Big(m(e, \hat{s}(e))\Big)}{u'\Big(m(e, \hat{s}(e))\Big)} \\
&\quad \times \Big\{p(e)\mathrm{E}_F\Big[u'\Big(\tilde{w} + R\hat{s}(e)\Big)\Big] + [1 - p(e)]\mathrm{E}_G\Big[u'\Big(\tilde{w} + R\hat{s}(e)\Big)\Big]\Big\}R \\
&= -u_2'\Big(\mu_G - p(e)\ell + R\hat{s}(e)\Big)R + \frac{u_2'\Big(m(e, \hat{s}(e))\Big)}{u'\Big(m(e, \hat{s}(e))\Big)} \\
&\quad \times \Big\{p(e)\mathrm{E}_F\Big[u'\Big(\tilde{w} + R\hat{s}(e)\Big)\Big] + [1 - p(e)]\mathrm{E}_G\Big[u'\Big(\tilde{w} + R\hat{s}(e)\Big)\Big]\Big\}R, \qquad (28)
\end{aligned}
$$

where the second equality follows from Eq. (24). Since $U(e, s)$ is concave for all $(e, s) \in [0, w_\circ] \times [0, w_\circ]$, Eq. (20) implies that $s(e) > (<)\ \hat{s}(e)$ if, and only if, $U_s(e, \hat{s}(e)) > (<)\ 0$.

Define the risk premia, $\pi_F(s)$ and $\pi_G(s)$, with respect to $F(w)$ and $G(w)$ for a fixed amount of saving, s, as the solutions to the following two equations:

$$
\mathrm{E}_F[u(\tilde{w} + Rs)] = u\Big(\mu_G - \ell + Rs - \pi_F(s)\Big), \qquad (29)
$$

and

$$
\mathrm{E}_G[u(\tilde{w} + Rs)] = u\Big(\mu_G + Rs - \pi_G(s)\Big), \qquad (30)
$$

respectively, where $0 \leq \pi_G(s) < \pi_F(s) + \ell$ since $G(w)$ dominates $F(w)$ via FSD. If $u(c_2)$ exhibits non-increasing absolute risk aversion, Eqs. (29) and (30) imply that

$$
\mathrm{E}_F[u'(\tilde{w} + Rs)] \geq u'\Big(\mu_G - \ell + Rs - \pi_F(s)\Big), \qquad (31)
$$

and

$$\mathrm{E}_G[u'(\tilde{w}+Rs)] \geq u'\Big(\mu_G + Rs - \pi_G(s)\Big), \tag{32}$$

respectively (Gollier, 2001; Kimball and Weil, 2009), where the inequalities hold as equalities if $u(c_2)$ exhibits constant absolute risk aversion.[21] Using Eqs. (29) and (30), we can write Eq. (19) as

$$p(e)u\Big(\mu_G - \ell + Rs - \pi_F(s)\Big) + [1-p(e)]u\Big(\mu_G + Rs - \pi_G(s)\Big) = u\Big(m(e,s)\Big). \tag{33}$$

If $u(c_2)$ exhibits non-increasing absolute risk aversion, Eq. (33) implies that

$$p(e)u'\Big(\mu_G - \ell + Rs - \pi_F(s)\Big) + [1-p(e)]u'\Big(\mu_G + Rs - \pi_G(s)\Big) \geq u'\Big(m(e,s)\Big). \tag{34}$$

It then follows from inequalities (31) and (32) that

$$\begin{aligned} & p(e)\mathrm{E}_F[u'(\tilde{w}+Rs)] + [1-p(e)]\mathrm{E}_G[u'(\tilde{w}+Rs)] \\ & \geq p(e)u'\Big(\mu_G - \ell + Rs - \pi_F(s)\Big) + [1-p(e)]u'\Big(\mu_G + Rs - \pi_G(s)\Big) \\ & \geq u'\Big(m(e,s)\Big), \end{aligned} \tag{35}$$

where the second inequality follows from inequality (34). Note that

$$\begin{aligned} & \frac{u_2'\Big(m(e,s)\Big)}{u'\Big(m(e,s)\Big)}\{p(e)\mathrm{E}_F[u'(\tilde{w}+Rs)] + [1-p(e)]\mathrm{E}_G[u'(\tilde{w}+Rs)]\} \\ & \geq u_2'\Big(m(e,s)\Big) \geq u_2'\Big(\mu_G - p(e)\ell + Rs\Big), \end{aligned} \tag{36}$$

where the first inequality follows from inequality (35), and the second inequality follows from the fact that $m(e,s) < \mu_G - p(e)\ell + Rs$ and $u_2''(c_2) \leq 0$. Setting $s = \hat{s}(e)$, it follows from Eq. (28) and inequality (36) that $U_e\Big(e, \hat{s}(e)\Big) \geq 0$ so that $s(e) \geq \hat{s}(e)$. Hence, the

[21]It is easily verified that non-increasing absolute risk aversion implies that $u'''(c_2) > 0$.

condition that $u(c_2)$ exhibits non-increasing absolute risk aversion is sufficient for prudence with Kreps-Porteus-Selden preferences independent of the Kreps-Porteus operator.

Indeed, the condition that $u(c_2)$ exhibits non-increasing absolute risk aversion is also necessary for prudence with Kreps-Porteus-Selden preferences independent of the Kreps-Porteus operator. To see this, consider the model of self-protection with $u_2(c_2) = c_2$. In this case, Eq. (28) becomes

$$U_s\Big(e, \hat{s}(e)\Big) = \frac{R}{u'\Big(m(e, \hat{s}(e))\Big)}\Big\{p(e)u'\Big(\mu_G - \ell + R\hat{s}(e)\Big)$$

$$+[1 - p(e)]u'\Big(\mu_G + R\hat{s}(e)\Big) - u'\Big(m(e, \hat{s}(e))\Big)\Big\} \geq 0, \tag{37}$$

since $s(e) \geq \hat{s}(e)$. It then follows from Eq. (19) that Eq. (37) implies that $u(c_2)$ exhibits non-increasing absolute risk aversion. Hence, we establish the following proposition (see also Gollier, 2001; Kimball and Weil, 2009).

Proposition 3. *Independent of the Kreps-Porteus operator, prudence with Kreps-Porteus-Selden preferences is equivalent to non-increasing absolute risk aversion with risk preferences.*

Independent of the Kreps-Porteus operator, $\phi(x)$, Proposition 3 shows that the necessary and sufficient condition under which the individual with Kreps-Porteus-Selden preferences is prudent hinges on the property of the von Neumann-Morgenstern utility function, $u(c_2)$. As pointed out by Kimball and Weil (2009), quadratic risk preferences do not in general lead to the absence of precautionary saving when one departs from the additive expected utility model. It is thus of interest to characterize prudence with Kreps-Porteus-Selden preferences when risk preferences are quadratic. In this case, Eq. (28) becomes

$$U_s\Big(e, \hat{s}(e)\Big) = u'\Big(\mu_G - p(e)\ell + R\hat{s}(e)\Big)R$$

$$\times\Bigg[\frac{u_2'\Big(m(e,\hat{s}(e))\Big)}{u'\Big(m(e,\hat{s}(e))\Big)} - \frac{u_2'\Big(\mu_G - p(e)\ell + R\hat{s}(e)\Big)}{u'\Big(\mu_G - p(e)\ell + R\hat{s}(e)\Big)}\Bigg], \tag{38}$$

where we have used the fact that $u(c_2)$ is quadratic. If the Kreps-Porteus operator, $\phi(x)$, is weakly concave, the adjustment factor, $u_2'(c_2)/u'(c_2)$, is non-increasing in c_2. Since $m(e,\hat{s}(e)) < \mu_G - p(e)\ell + R\hat{s}(e)$, the right-hand side of Eq. (38) is non-negative. Hence, we have $s(e) \geq \hat{s}(e)$.

To show that the weak concavity of $\phi(x)$ is also necessary for prudence with Kreps-Porteus-Selden preferences when risk preferences are quadratic, we suppose the contrary that there exists a non-empty interval, $[c_2^*, c_2^{**}]$, in which $u_2'(c_2)/u'(c_2)$ is increasing for all $c_2 \in [c_2^*, c_2^{**}]$, where $c_2^* < c_2^{**}$. Consider the model of self-protection and choose e, μ_G, and ℓ such that both $m(e,\hat{s}(e))$ and $\mu_G - p(e^\circ)\ell + R\hat{s}(e)$ are in $[c_2^*, c_2^{**}]$. In this case, the right-hand side of Eq. (38) is negative, which implies that $s(e) < \hat{s}(e)$, a contradiction. Hence, we establish the following proposition.

Proposition 4. *When risk preferences are quadratic, prudence with Kreps-Porteus-Selden preferences is equivalent to the weak concavity of the Kreps-Porteus operator.*

While Propositions 3 and 4 characterize prudence with Kreps-Porteus-Selden preferences, thereby precautionary saving, by imposing different restrictions on $u(c_2)$ and $\phi(x)$, the comparison between s^* and s° is not immediate because the underlying uncertainty is endogenously determined in our two-period model via the effort level exerted by the individual. As such, we need to take into account the fact that the optimal effort level under uncertainty, e^*, is not necessarily equal to that under certainty, e°.

To compare s^* with s°, we differentiate Eq. (18) with respect to s and evaluate the resulting derivative at $(e,s) = (e^*, s^\circ)$ to yield

$$U_s(e^*, s^\circ) = -u_1'(w_\circ - e^* - s^\circ) + \frac{u_2'\Big(m(e^*, s^\circ)\Big)}{u'\Big(m(e^*, s^\circ)\Big)}$$

$$\times\{p(e^*)\mathrm{E}_F[u'(\tilde{w}+Rs^\circ)]+[1-p(e^*)]\mathrm{E}_G[u'(\tilde{w}+Rs^\circ)]\}R$$

$$=u_1'(w_\circ-e^\circ-s^\circ)-u_1'(w_\circ-e^*-s^\circ)-u_2'\Big(\mu_G-p(e^\circ)\ell+Rs^\circ\Big)R$$

$$+\frac{u_2'\Big(m(e^*,s^\circ)\Big)}{u'\Big(m(e^*,s^\circ)\Big)}\{p(e^*)\mathrm{E}_F[u'(\tilde{w}+Rs^\circ)]+[1-p(e^*)]\mathrm{E}_G[u'(\tilde{w}+Rs^\circ)]\}R, \tag{39}$$

where the second equality follows from Eq. (24) with $e=e^\circ$ and $\hat{s}(e^\circ)=s^\circ$. Since $U(e,s)$ is concave for all $(e,s)\in[0,w_\circ]\times[0,w_\circ]$, Eq. (20) with $e=e^*$ implies that $s^*>(<)\ s^\circ$ if, and only if, $U_s(e^*,s^\circ)>(<)\ 0$.

If $e^*\le e^\circ$, it follows from $u_1''(c_1)<0$ that the sum of the first two terms on the right-hand side of Eq. (39) is non-negative. Given that the individual with Kreps-Porteus-Selden preferences is prudent independent of the Kreps-Porteus operator, $\phi(x)$, we can set $e=e^*$ and $s=s^\circ$ in inequality (36) to yield

$$\frac{u_2'\Big(m(e^*,s^\circ)\Big)}{u'\Big(m(e^*,s^\circ)\Big)}\{p(e^*)\mathrm{E}_F[u'(\tilde{w}+Rs^\circ)]+[1-p(e^*)]\mathrm{E}_G[u'(\tilde{w}+Rs^\circ)]\}$$

$$\ge u_2'\Big(m(e^*,s^\circ)\Big)\ge u_2'\Big(\mu_G-p(e^*)\ell+Rs^\circ\Big). \tag{40}$$

On the other hand, given that the individual with Kreps-Porteus-Selden preferences is prudent when risk preferences are quadratic, we have

$$\frac{u_2'\Big(m(e^*,s^\circ)\Big)}{u'\Big(m(e^*,s^\circ)\Big)}\{p(e^*)\mathrm{E}_F[u'(\tilde{w}+Rs^\circ)]+[1-p(e^*)]\mathrm{E}_G[u'(\tilde{w}+Rs^\circ)]\}$$

$$=\frac{u_2'\Big(m(e^*,s^\circ)\Big)}{u'\Big(m(e^*,s^\circ)\Big)}u'\Big(\mu_G-p(e^*)\ell+Rs^\circ\Big)\ge u_2'\Big(\mu_G-p(e^*)\ell+Rs^\circ\Big), \tag{41}$$

where the equality follows from the fact that $u(c_2)$ is quadratic, and the inequality follows from the fact that $u_2'(c_2)/u'(c_2)$ is non-increasing in c_2. If $e^*\le e^\circ$, $u_2''(c_2)\le 0$ implies that $u_2'\Big(\mu_G-p(e^*)\ell+Rs^\circ\Big)\ge u_2'\Big(\mu_G-p(e^\circ)\ell+Rs^\circ\Big)$. It then follows from either inequality (40) or inequality (41) that the sum of the last two terms on the right-hand side of Eq. (39) is non-negative. Hence, we conclude that $U_s(e^*, s^\circ) \geq 0$ if $e^* \leq e^\circ$, thereby rendering that $s^* \geq s^\circ$.

Indeed, $e^* \leq e^\circ$ is also necessary for $s^* \geq s^\circ$ under prudence with Kreps-Porteus-Selden preferences as characterized by either Proposition 3 or Proposition 4. To see this, consider first the case that the individual with Kreps-Porteus-Selden preferences is prudent independent of the Kreps-Porteus operator by setting $u_2(c_2) = c_2$ and letting $u(c_2)$ exhibit constant absolute risk aversion. In this case, Eq. (39) reduces to

$$U_s(e^*, s^\circ) = u_1'(w_\circ - e^\circ - s^\circ) - u_1'(w_\circ - e^* - s^\circ) \geq 0, \tag{42}$$

where the inequality follows from $s^* \geq s^\circ$. Since $u_1''(c_1) < 0$, Eq. (42) implies that $e^* \leq e^\circ$. Consider now the case that the individual with Kreps-Porteus-Selden preferences is prudent when risk preferences are quadratic by setting $u_2(c_2) = u(c_2)$ so that the Kreps-Porteus operator is linear. In this case, Eq. (39) also reduces to Eq. (42), implying that $e^* \leq e^\circ$. Hence, we establish the following proposition.

Proposition 5. *Given prudence with Kreps-Porteus-Selden preferences either independent of the Kreps-Porteus operator or with quadratic risk preferences, the individual optimally saves at least the same amount in the presence than in the absence of uncertainty, i.e.,* $s^* \geq s^\circ$*, if, and only if, the optimal effort level under certainty,* e°*, is no less than that under uncertainty,* e^**.*

The intuition for Proposition 5 is as follows. Since $u_1''(c_1) < 0$, the marginal cost of saving under uncertainty, $u_1'(w_\circ - e^* - s^\circ)$, does not exceed that under certainty, $u_1'(w_\circ - e^\circ - s^\circ)$, if, and only if, $e^* \leq e^\circ$. The expected marginal benefit of saving under uncertainty is given by

$$\frac{u_2'\Big(m(e^*, s^\circ)\Big)}{u'\Big(m(e^*, s^\circ)\Big)}\{p(e^*)\mathrm{E}_F[u'(\tilde{w} + Rs^\circ)] + [1 - p(e^*)]\mathrm{E}_G[u'(\tilde{w} + Rs^\circ)]\}R$$

$$\geq u_2'\Big(\mu_G - p(e^\circ)\ell + Rs^\circ\Big)R, \tag{43}$$

where the inequality follows from either inequality (40) or inequality (41) and $u_2'\Big(\mu_G - p(e^*)\ell + Rs^\circ\Big) \geq u_2'\Big(\mu_G - p(e^\circ)\ell + Rs^\circ\Big)$ given that $e^* \leq e^\circ$. When the individual with Kreps-Porteus-Selden preferences is prudent either independent of the Kreps-Porteus operator or with quadratic risk preferences, inequality (43) implies that the expected marginal benefit of saving under uncertainty is not smaller than that under certainty if, and only if, $e^* \leq e^\circ$. As such, effort and saving in our two-period model exhibit a kind of substitutability at the optimum so that the individual is induced to save at least the same amount in the presence than in the absence of uncertainty if, and only if, $e^* \leq e^\circ$.

4.2 Optimal effort and saving

To compare e° with the optimal effort level under uncertainty, e^*, we differentiate Eq. (18) with respect to e and evaluate the resulting derivative at $(e, s) = \Big(e^\circ, s(e^\circ)\Big)$ to yield

$$\begin{aligned}
U_e\Big(e^\circ, s(e^\circ)\Big) &= -u_1'\Big(w_\circ - e^\circ - s(e^\circ)\Big) - \frac{u_2'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)}{u'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)} \\
&\quad \times p'(e^\circ)\Big\{\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - \mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big]\Big\} \\
&= \frac{u_2'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)R}{u'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)}\Bigg\{\frac{\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - \mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big]}{\ell} \\
&\quad - p(e^\circ)\mathrm{E}_F\Big[u'\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - [1 - p(e^\circ)]\mathrm{E}_G\Big[u'\Big(\tilde{w} + Rs(e^\circ)\Big)\Big]\Bigg\},
\end{aligned} \tag{44}$$

where the second equality follows from Eq. (20) with $e = e^\circ$ and Eq. (27). Since $U(e, s)$ is concave for all $(e, s) \in [0, w_\circ] \times [0, w_\circ]$, Eq. (22) implies that $e^* < (>)\ e^\circ$ if, and only if, $U_e\Big(e^\circ, s(e^\circ)\Big) < (>)\ 0$.

When risk preferences are quadratic, Eq. (44) becomes[22]

$$U_e\Big(e^\circ, s(e^\circ)\Big) = \frac{u_2'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)R}{u'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)}$$

$$\times\Big[u'\Big(\mu_G - \frac{\ell}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell} + Rs(e^\circ)\Big) - u'\Big(\mu_G - p(e^\circ)\ell + Rs(e^\circ)\Big)\Big]. \quad (45)$$

The following proposition follows immediately from Eq. (45) and $u''(c_2) < 0$.

Proposition 6. *Given that the individual has Kreps-Porteus-Selden preferences with quadratic risk preferences and access to saving opportunities, the individual optimally exerts less (more) effort in the presence than in the absence of uncertainty, i.e.,* $e^* < (>)\ e^\circ$*, if, and only if, the optimal effort level under certainty,* e°*, satisfies that*

$$p(e^\circ) > (<)\ \frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}. \quad (46)$$

Since the individual has access to saving opportunities, effort is not used to smooth consumption. When risk preferences are quadratic, optimal effort is determined solely by the risk aversion incentive. It follows from Eq. (3) that $\sigma_H^2(e)$ is reduced by a decrease (an increase) in effort from e° if, and only if, e° satisfies condition (46), thereby rendering Proposition 6.

The following corollary is an immediate consequence of Propositions 5 and 6.

Corollary 1. *Given that the individual with Kreps-Porteus-Selden preferences is prudent when risk preferences are quadratic, the individual optimally exerts at most the same effort level and saves at least the same amount in the presence than in the absence of uncertainty, i.e.,* $e^* \leq e^\circ$ *and* $s^* \geq s^\circ$*, if, and only if, the optimal effort level under certainty,* e°*, satisfies*

[22] See footnote 14.

that

$$p(e^\circ) \geq \frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}. \tag{47}$$

Let $n(e,s)$ be the certainty equivalent of $\tilde{c}_2$ with respect to $u'(c_2)$, i.e., $n(e,s)$ is implicitly defined by

$$u'\Big(n(e,s)\Big) = p(e)\mathrm{E}_F[u'(\tilde{w} + Rs)] + [1 - p(e)]\mathrm{E}_G[u'(\tilde{w} + Rs)]. \tag{48}$$

Using Eq. (48) with $e = e^\circ$ and $s = s(e^\circ)$, we can write Eq. (44) as

$$U_e\Big(e^\circ, s(e^\circ)\Big) = \frac{u_2'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)R}{u'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)\ell}$$

$$\times\Big\{\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - \mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - u'\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)\ell\Big\}. \tag{49}$$

We derive sufficient conditions under which $e^* > (<)\ e^\circ$ in the following proposition.

Proposition 7. *Given that the individual has Kreps-Porteus-Selden preferences with prudent (imprudent) risk preferences and access to saving opportunities, the individual optimally exerts more (less) effort in the presence than in the absence of uncertainty, i.e.,* $e^* > (<)\ e^\circ$*, if the optimal effort level under certainty,* e°*, satisfies that*

$$n\Big(e^\circ, s(e^\circ)\Big) \geq (\leq)\ \mu_G - \frac{\ell}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell} + Rs(e^\circ). \tag{50}$$

Proof. See Appendix C. □

Since the individual has access to saving opportunities, effort is not used to smooth consumption. It follows from Eq. (48) that the expected marginal benefit of saving is equal to $u'\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)R$. On the other hand, using the derivation similar to the proof of Proposition 1, we can easily show that prudence (imprudence) and condition (12) ensure that the expected marginal benefit of effort satisfies that

$$-p'(e^{\circ})\Big\{\mathrm{E}_G\Big[u\Big(\tilde{w}+Rs(e^{\circ})\Big)\Big]-\mathrm{E}_F\Big[u\Big(\tilde{w}+Rs(e^{\circ})\Big)\Big]\Big\}$$

$$>(<)\ -u'\Big(\mu_G-p(e^{\circ})\ell+Rs(e^{\circ})\Big)p'(e^{\circ})\ell, \tag{51}$$

which is analogous to inequality (13). Since $u''(c_2)<0$, condition (50) ensures that

$$u'\Big(\mu_G-p(e^{\circ})\ell+Rs(e^{\circ})\Big)\geq(\leq)\ u'\Big(n\Big(e^{\circ},s(e^{\circ})\Big)\Big),$$

given that $p(e^{\circ})$ satisfies condition (12). The expected marginal benefit of effort as such is greater (smaller) than that of saving when the uncertainty prevails, thereby inducing the individual to exert more (less) effort than e°.

It is worth pointing out that condition (50) is stated in terms of $n\Big(e^{\circ},s(e^{\circ})\Big)$, which is implicitly defined by Eq. (48) with $e=e^{\circ}$ and $s=s(e^{\circ})$. This makes condition (50) less transparent and not readily applicable. As such, we offer more intuitive interpretation of condition (50) at the expense of more restrictive risk preferences in the following proposition.

Proposition 8. *Given that the individual has Kreps-Porteus-Selden preferences, wherein risk preferences satisfy non-decreasing (decreasing) absolute risk aversion and prudence, and access to saving opportunities, and that the risk premium,* π_G°*, which solves Eq. (30) at* $s=s(e^{\circ})$*, is sufficiently small such that* $\pi_G^{\circ}<\ell/2+(\sigma_F^2-\sigma_G^2)/2\ell$*, a sufficient (necessary) condition under which condition (50) holds so that the individual optimally exerts more effort in the presence than in the absence of uncertainty, i.e.,* $e^*>e^{\circ}$*, is that the optimal effort level under certainty,* e°*, satisfies that*

$$p(e^{\circ})\leq(<)\ \frac{\mathrm{E}_G\Big[u\Big(\tilde{w}+Rs(e^{\circ})\Big)\Big]-u\Big(\mu_G-\ell/2-(\sigma_F^2-\sigma_G^2)/2\ell+Rs(e^{\circ})\Big)}{\mathrm{E}_G\Big[u\Big(\tilde{w}+Rs(e^{\circ})\Big)\Big]-\mathrm{E}_F\Big[u\Big(\tilde{w}+Rs(e^{\circ})\Big)\Big]}$$

$$\in\left(0,\frac{1}{2}+\frac{\sigma_F^2-\sigma_G^2}{2\ell^2}\right). \tag{52}$$

Proof. See Appendix D. □

The condition, $\pi_G^\circ < \ell/2 + (\sigma_F^2 - \sigma_G^2)/2\ell$, ensures that the threshold in condition (52) is positive. This condition is satisfied trivially in the model of self-protection, which implies that $\pi_G^\circ = 0$. Indeed, this condition holds as long as μ_F and μ_G differ by a big enough margin or σ_G^2 is sufficiently small.

Under non-decreasing (decreasing) absolute risk aversion, inequality (35) implies that the expected marginal benefit of saving does not exceed (is greater than) $u'\Big(m\big(e^\circ, s(e^\circ)\big)\Big)R$. On the other hand, inequality (51) implies that the expected marginal benefit of effort exceeds $-u'\big(\mu_G - p(e^\circ)\ell + Rs(e^\circ)\big)p'(e^\circ)\ell$ given prudence and condition (12). Since $u''(c_2) < 0$, we have $u'\big(\mu_G - p(e^\circ)\ell + Rs(e^\circ)\big) < u'\Big(m\big(e^\circ, s(e^\circ)\big)\Big)$. To ensure that the expected marginal benefit of effort exceeds that of saving when the uncertainty prevails, i.e., condition (50) holds, $p(e^\circ)$ has to be bounded from above by a threshold that is strictly below $1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2$ to make the risk aversion incentive strong enough. As such, condition (52) is sufficient for condition (50) to hold when risk preferences satisfy non-decreasing absolute risk aversion. In the case of decreasing absolute risk aversion, $p(e^\circ)$ has to be subject to a even tighter upper bound to further strengthen the risk aversion incentive, thereby making condition (52) necessary but not sufficient for condition (50) to hold.

5. Conclusion

In this paper, we extend the one-period model of effort *à la* Crainich et al. (2016) to a two-period setting. In our generic model of effort in two periods, an individual exerts effort in monetary terms in the first period so as to improve risk in the second period. As in Crainich et al. (2016), we specify such a risk improvement by means of a linear combination of two fixed probability distribution functions that can be ranked via first-order stochastic dominance (FSD).[23] To disentangle the effect of risk preferences from that of time preferences on the individual's effort choice in an intertemporal setting, we adopt the recursive utility model of Kreps and Porteus (1978) and Selden (1978).

We conduct the comparative statics of effort with respect to the prevalence of uncertainty. Three interactive incentives, the precautionary incentive, risk aversion incentive, and consumption smoothing incentive, jointly determine whether the optimal effort level under uncertainty is higher or lower than that under certainty. A reasonable example is constructed to show that prudent individuals may optimally exert less effort when uncertainty prevails, which is consistent with recent experimental evidence for the negative relationship between prudence and self-protection regardless of the timing of loss (Masuda and Lee, 2019). To examine the precautionary motive of saving in our two-period model of effort, we characterize prudence with Kreps-Porteus-Selden preferences, which is defined by means of precautionary saving, by either restricting risk preferences to satisfy non-increasing absolute risk aversion (Gollier, 2001; Kimball and Weil, 2009) or focusing on preferences for late risk resolution when risk preferences are quadratic. Since the underlying uncertainty is endogenously determined via the choice of effort, the comparison between the optimal amount of saving under uncertainty and that under certainty is not immediate. A necessary and sufficient condition for prudence with Kreps-Porteus-Selden preferences to give rise to at least the same amount of saving when the endogenous uncertainty prevails is that the optimal effort level under uncertainty does not exceed that under certainty, thereby rendering that effort and saving are substitutes at the optimum.

While our generic model of effort is developed in a two-period setting, the introduction of saving opportunities makes the effort choice problem reduce to a static decision problem, owing to the dominance of saving over effort in achieving consumption smoothing between the two periods. As such, all the results concerning effort when endogenous saving is allowed are readily applicable to one-period models (Crainich et al., 2016; Peter, 2019) that can be analyzed in an analogous manner. Given that individuals care about not only consumption

[23] See Lee (2019) for a general risk improvement via FSD in a two-period model of self-protection.

or wealth but also some other attributes such as health, longevity, and environment, this calls for the use of multivariate utility functions that include these attributes in the arguments (Eeckhoudt et al., 2007; Liu and Menegatti, 2019), and for the incorporation of more than one source of uncertainty (Courbage et al., 2017) into the model. We leave all these interesting extensions for future research.

Appendix

A. Proof of Proposition 1

Consider first the case that $\phi(x)$ is weakly concave so that $-u_2(c_2)/u(c_2)$ is non-increasing in c_2. Since $m(e^\circ) < \mu_G - p(e^\circ)\ell$, Eq. (11) implies that

$$U'(e^\circ) \geq -\frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)} p'(e^\circ) \Big\{ \mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})] - u'\Big(\mu_G - p(e^\circ)\ell\Big)\ell \Big\}$$

$$= -\frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)} p'(e^\circ) \int_a^b \Big[u'(w) - u'\Big(\mu_G - p(e^\circ)\ell\Big)\Big][F(w) - G(w)]\mathrm{d}w, \tag{A.1}$$

where the equality follows from integration by parts. To sign the right-hand side of Eq. (A.1), we define the function of the tangent line to $u'(w)$ at the point, $w = \mu_G - p(e^\circ)\ell$:

$$T(w) = u'\Big(\mu_G - p(e^\circ)\ell\Big) + u''\Big(\mu_G - p(e^\circ)\ell\Big)[w - \mu_G + p(e^\circ)\ell], \tag{A.2}$$

for all $w \in [a, b]$. Since $u'''(c_2) > 0$, Eq. (A.2) implies that $u'(w) > T(w)$ for all $w \neq \mu_G - p(e^\circ)\ell$. Using Eq. (A.2) and $F(w) \geq G(w)$ for all $w \in [a, b]$, we notice that the right-hand side of Eq. (A.1) must exceed

$$-\frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)} p'(e^\circ) \int_a^b u''\Big(\mu_G - p(e^\circ)\ell\Big)[w - \mu_G + p(e^\circ)\ell][F(w) - G(w)]\mathrm{d}w$$

$$= \frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)} p'(e^\circ) u''\Big(\mu_G - p(e^\circ)\ell\Big) \int_a^b \Big\{[\mu_G - p(e^\circ)\ell]w - \frac{w^2}{2}\Big\} \mathrm{d}[G(w) - F(w)]$$

$$= \frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)} p'(e^\circ) u''\Big(\mu_G - p(e^\circ)\ell\Big) \Big\{\Big[\frac{1}{2} - p(e^\circ)\Big]\ell^2 + \frac{\sigma_F^2 - \sigma_G^2}{2}\Big\}, \tag{A.3}$$

where the first equality follows from integration by parts. If e° satisfies condition (12), the right-hand side of Eq. (A.3) is non-negative so that the right-hand side of Eq. (A.1) is positive, thereby rendering that $e^* > e^\circ$.

Consider now the case that $\phi''(x) \geq 0$ so that $-u_2(c_2)/u(c_2)$ is non-decreasing in c_2. Since $m(e^\circ) < \mu_G - p(e^\circ)\ell$, Eq. (11) implies that

$$U'(e^\circ) \leq -\frac{u_2'\Big(\mu_G - p(e^\circ)\ell\Big)}{u'\Big(\mu_G - p(e^\circ)\ell\Big)} p'(e^\circ) \Big\{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})] - u'\Big(\mu_G - p(e^\circ)\ell\Big)\ell\Big\}$$

$$= -\frac{u_2'\Big(\mu_G - p(e^\circ)\ell\Big)}{u'\Big(\mu_G - p(e^\circ)\ell\Big)} p'(e^\circ) \int_a^b \Big[u'(w) - u'\Big(\mu_G - p(e^\circ)\ell\Big)\Big][F(w) - G(w)]\mathrm{d}w. \tag{A.4}$$

Since $u'''(c_2) < 0$, Eq. (A.2) implies that $u'(w) < T(w)$ for all $w \neq \mu_G - p(e^\circ)\ell$. Using Eq. (A.2), we notice that the right-hand side of Eq. (A.4) must be less than

$$-\frac{u_2'\Big(\mu_G - p(e^\circ)\ell\Big)}{u'\Big(\mu_G - p(e^\circ)\ell\Big)} p'(e^\circ) \int_a^b u''\Big(\mu_1 - p(e^\circ)\ell\Big)[w - \mu_G + p(e^\circ)\ell][F(w) - G(w)]\mathrm{d}w$$

$$= \frac{u_2'\Big(\mu_G - p(e^\circ)\ell\Big)}{u'\Big(\mu_G - p(e^\circ)\ell\Big)} p'(e^\circ) u''\Big(\mu_1 - p(e^\circ)\ell\Big) \int_a^b \Big\{[\mu_G - p(e^\circ)\ell]w - \frac{w^2}{2}\Big\} \mathrm{d}[G(w) - F(w)]$$

$$= \frac{u_2'\Big(\mu_G - p(e^\circ)\ell\Big)}{u'\Big(\mu_G - p(e^\circ)\ell\Big)} p'(e^\circ) u''\Big(\mu_G - p(e^\circ)\ell\Big) \Big\{\Big[\frac{1}{2} - p(e^\circ)\Big]\ell^2 + \frac{\sigma_F^2 - \sigma_G^2}{2}\Big\}, \tag{A.5}$$

where the first equality follows from integration by parts. If e° satisfies condition (12), the right-hand side of Eq. (A.3) is non-positive so that the right-hand side of Eq. (A.4) is negative, thereby rendering that $e^* < e^\circ$.

B. Proof of Proposition 2

Define the following function:

$$z(p) = u^{-1}\Big(p\mathrm{E}_F[u(\tilde{w})] + (1-p)\mathrm{E}_G[u(\tilde{w})]\Big). \tag{A.6}$$

It is evident from Eqs. (8) and (A.6) that $z\Big(p(e^\circ)\Big) = m(e^\circ)$. Eq. (A.6) implies that $z(0) = u^{-1}\Big(\mathrm{E}_G[u(\tilde{w})]\Big) = \mu_G - \pi_G$, $z(1) = u^{-1}\Big(\mathrm{E}_F[u(\tilde{w})]\Big) = \mu_F - \pi_F$, and

$$z'(p) = -\frac{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})]}{u'\Big(z(p)\Big)} < 0, \tag{A.7}$$

for all $p \in [0,1]$, where $\pi_G \geq 0$ and $\pi_F \geq 0$ are the risk premia with respect to $G(w)$ and $F(w)$, respectively. Using Eq. (A.6) and setting $p = 1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2$ yields

$$\begin{aligned} u\left(z\left(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\right)\right) &= \left(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\right)\mathrm{E}_F[u(\tilde{w})] + \left(\frac{1}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\right)\mathrm{E}_G[u(\tilde{w})] \\ &< \left(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\right)u(\mu_G - \ell) + \left(\frac{1}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\right)u(\mu_G) \\ &< u\left(\mu_G - \frac{\ell}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell}\right), \end{aligned} \tag{A.8}$$

where the inequalities follow from $u''(c_2) < 0$ and Jensen's inequality. Hence, Eq. (A.8) implies that

$$z\left(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\right) < \mu_G - \frac{\ell}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell}. \tag{A.9}$$

If $z(0) > \mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell$, i.e., $\ell/2 + (\sigma_F^2 - \sigma_G^2)/2\ell > \pi_G$, it follows from Eq. (A.7) that there must exist a unique point, $p^* \in (0, 1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2)$, at which $z(p^*) = \mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell$. Using Eq. (A.6) to solve for p^* yields

$$p^* = \frac{\mathrm{E}_G[u(\tilde{w})] - u\Big(\mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell\Big)}{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})]}. \tag{A.10}$$

Since $m(e^\circ) < \mu_G - p(e^\circ)\ell$ and $u_2''(c_2) < 0$, Eq. (11) implies that

$$U'(e^\circ) > -\frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)}p'(e^\circ)\Big\{\mathrm{E}_G[u(\tilde{w})] - \mathrm{E}_F[u(\tilde{w})] - u'\Big(m(e^\circ)\Big)\ell\Big\}$$

$$= -\frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)}p'(e^\circ)\int_a^b \Big[u'(w) - u'\Big(m(e^\circ)\Big)\Big][F(w) - G(w)]\mathrm{d}w, \qquad \text{(A.11)}$$

where the equality follows from integration by parts. To sign the right-hand side of Eq. (A.11), we define the function of the tangent line to $u'(w)$ at the point, $w = m(e^\circ)$:

$$T_m(w) = u'\Big(m(e^\circ)\Big) + u''\Big(m(e^\circ)\Big)[w - m(e^\circ)], \qquad \text{(A.12)}$$

for all $w \in [a, b]$. Since $u'''(c_2) > 0$, Eq. (A.12) implies that $u'(w) > T_m(w)$ for all $w \neq m(e^\circ)$. Using Eq. (A.12) and $F(w) \geq G(w)$ for all $w \in [a, b]$, we notice that the right-hand side of Eq. (A.11) must exceed

$$-\frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)}p'(e^\circ)\int_a^b u''\Big(m(e^\circ)\Big)[w - m(e^\circ)][F(w) - G(w)]\mathrm{d}w$$

$$= \frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)}p'(e^\circ)u''\Big(m(e^\circ)\Big)\int_a^b \left[m(e^\circ)w - \frac{w^2}{2}\right]\mathrm{d}[G(w) - F(w)]$$

$$= \frac{u_2'\Big(m(e^\circ)\Big)}{u'\Big(m(e^\circ)\Big)}p'(e^\circ)u''\Big(m(e^\circ)\Big)\left\{\left[m(e^\circ) - \left(\mu_G - \frac{\ell}{2}\right)\right]\ell + \frac{\sigma_F^2 - \sigma_G^2}{2}\right\}, \qquad \text{(A.13)}$$

where the first equality follows from integration by parts. Since $p(e^\circ) \leq p^*$, we have $z\Big(p(e^\circ)\Big) \geq z(p^*)$ and $m(e^\circ) \geq \mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell$. In this case, the right-hand side of Eq. (A.13) is non-negative so that the right-hand side of Eq. (A.11) is positive, thereby rendering that $e^* > e^\circ$.

C. Proof of Proposition 7

Using integration by parts, we can write inequality (49) as

$$U_e\Big(e^\circ, s(e^\circ)\Big) = \frac{u_2'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)R}{u'\Big(m\Big(e^\circ, s(e^\circ)\Big)\Big)\ell}$$

$$\times \int_a^b \Big[u'\Big(w + Rs(e^\circ)\Big) - u'\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)\Big][F(w) - G(w)]\mathrm{d}w. \quad \text{(A.14)}$$

To sign the right-hand side of Eq. (A.14), we define the function of the tangent line to $u'\Big(w + Rs(e^\circ)\Big)$ at the point, $w + Rs(e^\circ) = n\Big(e^\circ, s(e^\circ)\Big)$:

$$T_n\Big(w + Rs(e^\circ)\Big) = u'\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)$$

$$+u''\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)\Big[w + Rs(e^\circ) - n\Big(e^\circ, s(e^\circ)\Big)\Big], \quad \text{(A.15)}$$

for all $w \in [a, b]$. Since $u'''(c_2) > (<)\ 0$, Eq. (A.15) implies that $u'\Big(w + Rs(e^\circ)\Big) > (<)\ T_n\Big(w + Rs(e^\circ)\Big)$ for all $w + Rs(e^\circ) \neq n\Big(e^\circ, s(e^\circ)\Big)$. Using Eq. (A.15) and $F(w) \geq G(w)$ for all $w \in [a, b]$, we notice that the integral on the right-hand side of Eq. (A.14) must be larger (smaller) than

$$\int_a^b u''\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)\Big[w + Rs(e^\circ) - n\Big(e^\circ, s(e^\circ)\Big)\Big][F(w) - G(w)]\mathrm{d}w$$

$$= -u''\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)\int_a^b \Big\{\Big[n\Big(e^\circ, s(e^\circ)\Big) - Rs(e^\circ)\Big]w - \frac{w^2}{2}\Big\}\mathrm{d}[G(w) - F(w)]$$

$$= -u''\Big(n\Big(e^\circ, s(e^\circ)\Big)\Big)\Big\{\Big[n\Big(e^\circ, s(e^\circ)\Big) - Rs(e^\circ) - \Big(\mu_G - \frac{\ell}{2}\Big)\Big]\ell + \frac{\sigma_F^2 - \sigma_G^2}{2}\Big\}, \quad \text{(A.16)}$$

where the first equality follows from integration by parts. Condition (50) ensures that the right-hand side of Eq. (A.16) is non-negative (non-positive) so that the right-hand side of Eq. (A.14) is positive (negative), thereby rendering that $e^* > (<)\ e^\circ$.

D. Proof of Proposition 8

Define the following function:

$$\hat{z}(p) = u^{-1}\Big(p\mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] + (1-p)\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big]\Big). \tag{A.17}$$

Eq. (A.17) implies that $\hat{z}(0) = \mu_G + Rs(e^\circ) - \pi_G^\circ$, $\hat{z}(1) = \mu_G - \ell + Rs(e^\circ) - \pi_F^\circ$, and

$$\hat{z}'(p) = -\frac{\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - \mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big]}{u'\Big(\hat{z}(p)\Big)} < 0, \tag{A.18}$$

for all $p \in [0,1]$, where $\pi_F^\circ \geq 0$ and $\pi_G^\circ \geq 0$ are the risk premia defined in Eqs. (29) and (30) at $s = s(e^\circ)$, respectively. Using Eq. (A.17) and setting $p = 1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2$ yields

$$\begin{aligned}
&u\bigg(\hat{z}\bigg(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\bigg)\bigg) \\
&= \bigg(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\bigg)\mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] + \bigg(\frac{1}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\bigg)\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] \\
&< \bigg(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\bigg)u\Big(\mu_G - \ell + Rs(e^\circ)\Big) + \bigg(\frac{1}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\bigg)u\Big(\mu_G + Rs(e^\circ)\Big) \\
&< u\bigg(\mu_G - \frac{\ell}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell} + Rs(e^\circ)\bigg),
\end{aligned} \tag{A.19}$$

where the inequalities follow from $u''(c_2) < 0$ and Jensen's inequality. Hence, Eq. (A.19) implies that

$$\hat{z}\bigg(\frac{1}{2} + \frac{\sigma_F^2 - \sigma_G^2}{2\ell^2}\bigg) < \mu_G - \frac{\ell}{2} - \frac{\sigma_F^2 - \sigma_G^2}{2\ell} + Rs(e^\circ). \tag{A.20}$$

If $\hat{z}(0) > \mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell + Rs(e^\circ)$, i.e., $\ell/2 + (\sigma_F^2 - \sigma_G^2)/2\ell > \pi_G^\circ$, it follows from Eq. (A.18) that there must exist a unique point, $\hat{p}^* \in (0, 1/2 + (\sigma_F^2 - \sigma_G^2)/2\ell^2)$, at which $\hat{z}(\hat{p}^*) = \mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell + Rs(e^\circ)$. Using Eq. (A.17) to solve for $\hat{p}^*$ yields

$$\hat{p}^* = \frac{\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - u\Big(\mu_G - \ell/2 - (\sigma_F^2 - \sigma_G^2)/2\ell + Rs(e^\circ)\Big)}{\mathrm{E}_G\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big] - \mathrm{E}_F\Big[u\Big(\tilde{w} + Rs(e^\circ)\Big)\Big]}. \tag{A.21}$$

Let $\hat{p}^n \in (0,1)$ be the solution to $\hat{z}(\hat{p}^n) = n\Big(e^\circ, s(e^\circ)\Big)$. It follows from Eq. (A.18) that condition (50) is equivalent to $\hat{p}^n \leq \hat{p}^*$.

It is evident from Eqs. (19) and (A.17) that $\hat{z}\Big(p(e^\circ)\Big) = m\Big(e^\circ, s(e^\circ)\Big)$. Under non-decreasing (decreasing) absolute risk aversion, inequality (35) becomes

$$p(e)\mathrm{E}_F[u'(\tilde{w} + Rs)] + [1 - p(e)]\mathrm{E}_G[u'(\tilde{w} + Rs)] \leq (>)\ u'\Big(m(e,s)\Big),$$

so that $n(e,s) \geq (<)\ m(e,s)$ from $u''(c_2) < 0$ and Eq. (48). It follows from Eq. (A.18) that $\hat{p}^n \leq (>)\ p(e^\circ)$ under non-decreasing (decreasing) absolute risk aversion. Hence, under non-decreasing absolute risk aversion, $p(e^\circ) \leq \hat{p}^*$ implies that $\hat{p}^n \leq \hat{p}^*$ so that condition (52) is sufficient for condition (50). On the other hand, under decreasing absolute risk aversion, $\hat{p}^n \leq \hat{p}^*$ implies that $p(e^\circ) < \hat{p}^*$ so that condition (52) is necessary for condition (50).

E. Second-order conditions for optimal effort and saving

Let $p = p(e)$, $u_1'' = u_1''(w_\circ - e - s)$, $u_F = \mathrm{E}_F[u(\tilde{w} + Rs)]$, $u_F' = \mathrm{E}_F[u'(\tilde{w} + Rs)]$, $u_F'' = \mathrm{E}_F[u''(\tilde{w} + Rs)]$, $u_G = \mathrm{E}_G[u(\tilde{w} + Rs)]$, $u_G' = \mathrm{E}_G[u'(\tilde{w} + Rs)]$, $u_G'' = \mathrm{E}_G[u''(\tilde{w} + Rs)]$, $\alpha = u_2'\Big(m(e,s)\Big)/u'\Big(m(e,s)\Big)$, and

$$\beta = \frac{u_2'\Big(m(e,s)\Big)}{u'\Big(m(e,s)\Big)^2}\Bigg[\frac{u''\Big(m(e,s)\Big)}{u'\Big(m(e,s)\Big)} - \frac{u_2''\Big(m(e,s)\Big)}{u_2'\Big(m(e,s)\Big)}\Bigg].$$

To ensure that $U(e,s)$ is concave for all $(e,s) \in [0, w_\circ] \times [0, w_\circ]$, we require that $U_{ss}(e,s) < 0$, $U_{ee}(e,s) < 0$, and $U_{ss}(e,s)U_{ee}(e,s) - U_{es}(e,s)^2 > 0$, where

$$U_{ss}(e,s) = u_1'' - \beta[pu_F' + (1-p)u_G']^2R^2 + \alpha[pu_F'' + (1-p)u_G'']R^2,$$

$$U_{ee}(e,s) = u_1'' - \beta(u_G - u_F)^2 p'^2 - \alpha(u_G - u_F)p'',$$

and

$$U_{es}(e,s) = u_1'' + \beta[pu_F' + (1-p)u_G'](u_G - u_F)p'R + \alpha(u_F' - u_G')p'R.$$

If $\phi''(x) \le 0$, we have $\beta \ge 0$. Furthermore, if $p''(e) \ge 0$, we have $U_{ss}(e,s) < 0$, $U_{ee}(e,s) < 0$, and $U_{es}(e,s) < 0$. Note that

$$
\begin{aligned}
&U_{ss}(e,s)U_{ee}(e,s) - U_{es}(e,s)^2 \\
&= -\beta u_1''\{(u_G - u_F)p' + [pu_F' + (1-p)u_G']R\}^2 \\
&\quad +\alpha\beta(u_G - u_F)\{[pu_F' + (1-p)u_G']p''u_G' - (u_G - u_F)[pu_F'' + (1-p)u_G'']p^{'2}\}R^2 \\
&\quad +\alpha\beta(u_G - u_F)(u_F' - u_G')[pu_F' + (1-p)u_G'](p''p - 2p^{'2})R^2 \\
&\quad +\alpha u_1''[pu_F'' + (1-p)u_G'']R^2 - \alpha u_1''[(u_G - u_F)p'' + 2(u_F' - u_G')p'R] \\
&\quad -\alpha^2\{(u_G - u_F)[pu_F'' + (1-p)u_G'']p'' + (u_F' - u_G')^2 p^{'2}\}R^2.
\end{aligned}
$$

The first two terms are positive. The third term is positive if $p''p > 2p^{'2}$. A necessary condition for this to hold is that the technical rate of return on effort, $-p'/p$, is decreasing (Courbage et al. 2017). The fourth term is positive. Following Hofmann and Peter (2016), we assume that

$$
-\frac{u_F''}{u_F'} < \frac{1}{2R}\left(\frac{u_F'}{u_G'}\right)\left(-\frac{p''}{p'}\right),
$$

i.e., absolute risk aversion in the loss state is sufficiently small, which implies that $(u_G - u_F)p'' > -2(u_F' - u_G')p'R$ (see Hofmann and Peter, 2016), so that the fifth term is positive. If absolute risk aversion decreases at a sufficient rate in that

$$
-\frac{u_G'''}{u_G''} > -\left(\frac{u_F''}{u_G''}\right)^2 \frac{u_G''}{u_G'},
$$

which implies that $(u_G - u_F)(u_G'' - u_F'') > (u_F' - u_G')^2$ (see Hofmann and Peter, 2016). Together with $p''p > 2p^{'2}$, the last term is positive. Hence, the set of sufficient conditions that guarantee the concavity of $U(e,s)$ is non-trivial. For the ease of exposition, we simply assume that $U(e,s)$ is concave for all $(e,s) \in [0, w_\circ] \times [0, w_\circ]$.

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