Foreign Direct Investment and Debt Financing in Emerging Economies

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Abstract

This paper analyzes the dynamic pattern of capital financing in emerging economies over the business cycle. We show empirically that in normal periods, FDI and external debt financing are procyclical while, during crises, FDI is countercyclical whereas external debt remains procyclical. We then build a small open economy model with borrowing constraints and technology spillovers from foreign multinationals to explain the pattern of capital inflows to emerging economies. Our calibrated model generates procyclical FDI and debt following a productivity shock, and a large fall in debt financing and a small positive change in FDI following a financial shock consistent with our empirical observations on emerging economies.

Keywords: Financial frictions, FDI, debt financing, financial crisis

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1. Introduction

This paper studies business cycle pattern of foreign direct investment (henceforth FDI) and external debt inflows into emerging economies. During the past few decades, a wave of financial openness has swept emerging economies and induced large foreign capital inflows. Many papers have documented the dynamic pattern and composition of capital inflows (Aguiar and Gopinath (2005), Smith and Valderrama (2009), Broner et al. (2013), and Alquist et al. (2016)). In this paper we focus on the starkly different behavior of capital financing in normal times and during financial crises.

We begin in Section 2 by presenting stylized facts about the dynamics of FDI and external debt financing in emerging economies for the period 1980-2015. A key feature is that the cyclical pattern of foreign direct investment is different in normal periods and crises times: in normal periods, FDI and external debt financing are both procyclical; during financial crises, external debt financing is still procyclical but FDI becomes slightly countercyclical. Examples of this feature can be found in the 1990s financial crises in Asia and Latin America. Following these, external debt financing turned negative in crisis-affected emerging economies. However, unlike foreign bank lending and portfolio investment, FDI remained positive and kept adding to the FDI stock (Athukorala (2003)).

Krugman (2000) proposes a 'fire-sale' channel to account for such a pattern. He argues that FDI inflows in the crisis-affected countries increase because the costs of establishing and expanding production facilities in those countries will decrease following a financial crisis. The decrease in costs comes from a depreciation of local currencies and lower property prices because of fire sales, given the heavy indebtedness and limited access to credit of local firms. International multinational enterprises (henceforth MNCs) may see this as an opportunity to establish settlement and expand market share in the crisis-affected countries. As a result, FDI in the short and medium term may increase after the crisis. Given MNCs have already set up international connections, they are better shielded from adverse local shocks. They also presumably have greater access than local firms to credit, distribution channels and market information. As a result, they are better equipped to smooth out any economic disruption caused by adverse shocks (Lipsey (2001)).

In this paper, we incorporate the above fire-sale channel and other realistic features of MNC-owned firms in an otherwise standard small open economy real business cycle model to study the comovement pattern of FDI and external debt. In the model, two production sectors, namely domestic firms and MNC-owned firms produce the same final good. These firms are different in two respects. First, firms are subject to borrowing constraints following Gertler and Karadi

(2011). With this assumption, we depart from Modigliani and Miller (1958), so different forms of financing are not perfectly substitutable and various types of capital flows at the country level behave differently at different stages of the business cycle. To reflect the fact that MNC-owned firms have better access to international financial markets than domestic firms, we assume MNC-owned firms face a looser borrowing constraint.

Second, we assume that MNC-owned firms are more productive than domestic firms. When a domestic firm is acquired by an MNC, the firm may gain access to better production technology (Alfaro and Charlton (2013)). Foreign investors may also bring in better institution or governance (Chari et al. (2009)). There is also strong evidence (Aitken and Harrison (1999)) supporting a knowledge spillover channel.

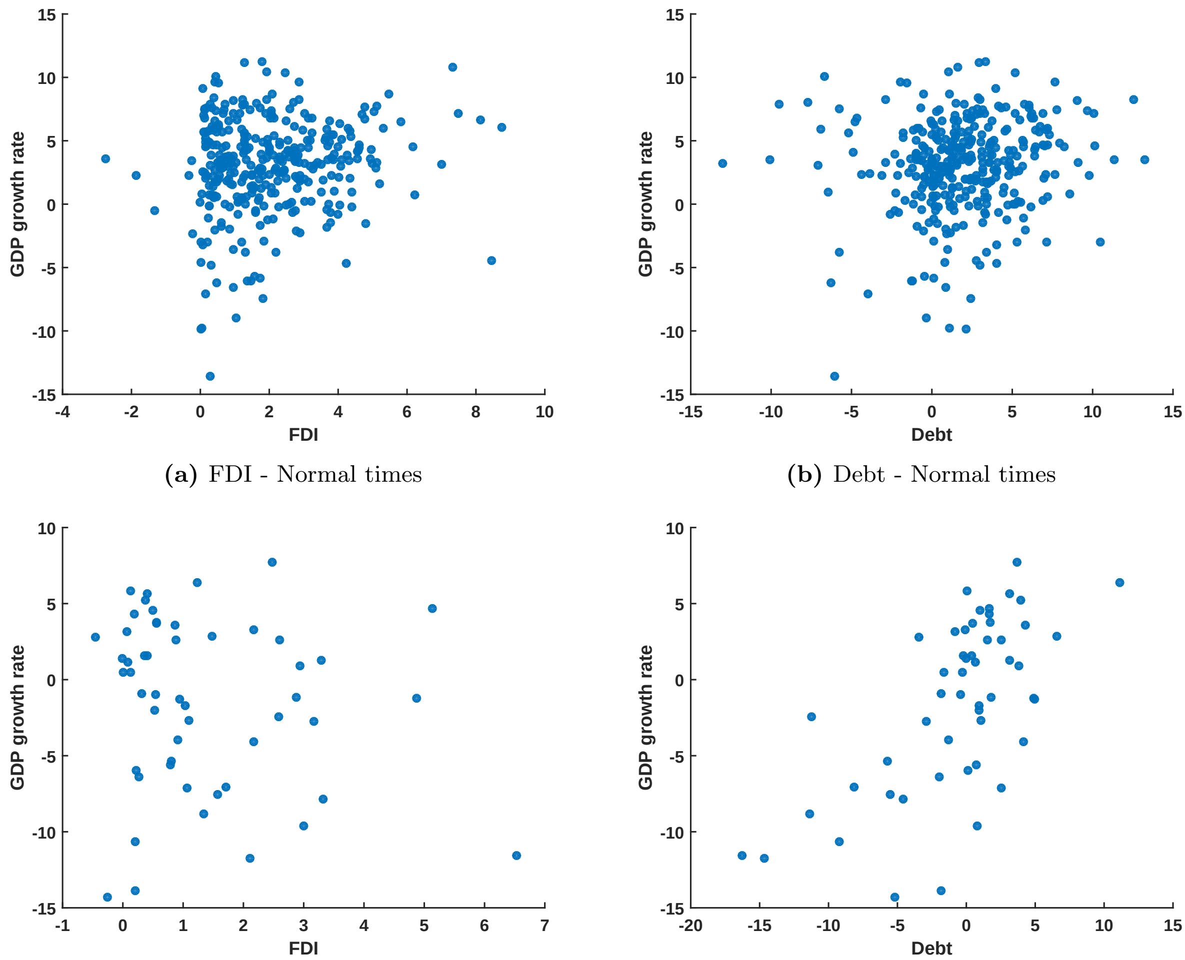
Given these distinctions, an MNC-owned firm has a larger value than a domestically-owned firm, everything else equal. The valuation difference creates flows of FDI. Once a local firm is acquired by a foreign multinational, its operation is more productive and is less constrained by financial frictions. Therefore, foreign multinationals can achieve a higher value of local firms. The wedge gives foreign multinationals incentives to acquire domestic firms. The acquisition price, or FDI inflow, is determined by Nash-bargaining which splits the difference in valuation of the firm between a domestic seller and an international buyer.

In this framework, the borrowing constraint is modelled as a stochastic process to mimic a financial shock during a crisis following Jermann and Quadrini (2012). A negative financial shock tightens the constraint faced by domestic firms, increases the fraction of value of the firm that can be diverted, depresses the value of the firm to domestic households, and enlarges the valuation wedge between a domestic firm and an MNC-owned firm. As a consequence, domestically-owned firms borrow less and FDI increases. On the other hand, a negative productivity shock impairs the balance sheets of domestically-owned firms, reduces the size of acquired firms, and leads to a fall in FDI. We calibrate our model with emerging market data and show that it can generate procyclical FDI and external debt in normal times when financial shocks are muted, and countercyclical FDI in crises times when financial shocks are turned on.

There is a commonly expressed notion that foreign firms acquire assets abroad at cheap prices during periods of weak currency. The anticipation of a future currency appreciation may trigger foreign investors to conduct more direct investment activities in crisis-hit economies. To address this potential channel, we extend our model to include an endogenous exchange rate adjustment path and portfolio choice by MNC-owned firms. To do so, we introduce importing and exporting activities as well as domestic and international lending. We find that expected real exchange rate appreciation increases the foreign investors' valuation gap directly, therefore increasing the attractiveness of acquisitions. Exchange rate adjustments, however, may decrease the value of FDI by reducing the size of acquired firms, especially when they rely heavily on imported intermediate goods. We find that the overall effect of a financial shock to FDI in an economy with exchange rate adjustments is still positive. Hence, our main results are robust to the inclusion of an exchange rate appreciation channel.

Our analysis is related to a large and growing literature on capital flows across countries. Ju and Wei (2010) provides a two-country model to study two-way capital flows with a focus on corporate governance and property rights. Wang et al. (2017) explains the two-way capital flow pattern between China and US using credit frictions. Mendoza et al. (2009) attributes the difference in a country's financial portfolio to differences in financial development. However, the bulk of this literature focuses on the long-run determinants rather than the cyclical pattern of capital flows. The paper closest in spirit to ours is Smith and Valderrama (2009), which studies the comovement between FDI and debt in a small open economy setup with costly debt financing. However, our paper differs from Smith and Valderrama (2009) in several aspects. First, our analysis reveals that the composition of capital financing differs between normal period and episodes of financial crises. Therefore, we can better account for the behavior of capital flows by capturing the heterogeneity in the responses of FDI and debt to different shocks over the business cycle. Second, Smith and Valderrama (2009) assume reduced form adjustment costs when raising debt in international markets; whereas we model financial frictions by explicitly considering an enforcement problem between borrowers and lenders, which allows us to study financial shocks in a non-trivial manner. Third, we show that, under our model setup, a firm's value has an analytical solution, which allows us to study a larger model with richer dynamics.

The rest of the paper is organized as follows. Section two provides empirical evidence. Section three presents the model. Section four describes calibration of the model. Section five analyzes the model properties and quantitative results. Section six extends the benchmark model to include the real exchange rate and portfolio choice between domestic and foreign debt. Section seven concludes.



(a) FDI - Normal times

(b) Debt - Normal times

(c) FDI - Crisis times

(d) Debt - Crisis times

Fig. 1. Capital inflows and GDP growth. *Note*: All flows are gross inflows as a percent of GDP. FDI refers to foreign direct investment inflows. Debt refers to both portfolio debt and other debt instruments inflows as in Balance of Payments. Annual data from 1980 to 2015. Countries included: Argentina, Brazil, Colombia, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Thailand, and Turkey. Source: World Bank and Alfaro et al. (2014).

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2. Empirical evidence

We first present stylized facts concerning FDI and external debt inflows to emerging economies. Our sample contains annual data for 1980-2015 for eleven emerging economies including Argentina, Brazil, Colombia, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Thailand, and Turkey. FDI is measured as FDI capital inflows as a percent of GDP. External debt financing is measured as portfolio debt and other debt instrument inflows as a percent of GDP. In order to analyze capital inflows around crises, we use the Broner et al. (2013) indicator to capture the beginning of a crisis on an annual basis.[1] That covers all major banking, currency, and debt crises in the history of emerging market economies. According to their crisis definition, each country has at least two crises in our sample period, and there are in total 52 crisis-year observations. A detailed list of the year of these is provided in Appendix E .

Figure 1 plots both variables against the real per capita GDP growth rate. This figure shows a positive comovement between the growth of output and FDI and debt inflows during normal times, which indicates that capital inflows in the form of FDI and external debt tend to move in tandem with the performance of the domestic economy in normal times. However, this correlation seems not to hold during crisis years, when a retrenchment in debt inflows is observed. We see a negative correlation between FDI inflows and the GDP growth rate, implying surprisingly stable FDI inflows into emerging market economies during crises.

A more formal regression analysis confirms the findings above. We consider the following regression:

$$Y_{i,t} = \beta_0 + \beta_1 \widetilde{\text{gdp}}_{i,t} + \beta_2 \left(\widetilde{\text{gdp}}_{i,t} \times D_{i,t} \right) + \gamma' X_{i,t} + I_i + \epsilon_{i,t}. \quad (1)$$

The dependent variable is either FDI or external debt inflows to GDP ratio. We use the domestic per capita GDP growth rate $\widetilde{\text{gdp}}_{i,t}$ to capture the stage of business cycles of emerging economy i at year t. The crisis dummy, $D_{i,t}$, equals 1 when there is a crisis and 0 otherwise. The coefficients β_1 and β_2 capture the comovement between the two types of capital flows and output during normal and crises times respectively. $X_{i,t}$ represents a set of control variables including the growth rate of per capita GDP of the US (which proxies global economic conditions), the exchange rate regime (which proxies the monetary policy framework), and the Chinn and Ito (2008) index (which captures the degree of capital controls).[2] We also include country fixed effects reflecting economic

[1] We conduct robustness checks and confirm that our empirical results are robust to alternative crisis definitions. Details can be provided from the authors.

[2] We use the annual fine exchange rate regime classification by Ilzetzki et al. (2017) for countries in our sample.

and political structures and other relevant features that are potentially important for explaining cross-country difference.

Table 1
Cyclicality of capital inflows.

	(1) FDI	(2) FDI	(3) FDI	(4) Debt	(5) Debt	(6) Debt
GDP Growth	0.0732***	0.0966***	0.0690***	0.155***	0.189***	0.123**
	(0.0233)	(0.0209)	(0.0210)	(0.0505)	(0.0538)	(0.0571)
GDP Growth×Crisis Dummy	-0.0902*	-0.105**	-0.0997**	0.492***	0.411***	0.334***
(Interactive Term)	(0.0477)	(0.0411)	(0.0391)	(0.103)	(0.106)	(0.106)
US GDP Growth			0.0218			-0.185**
			(0.0336)			(0.0913)
Exchange Rate Regime			0.0280			-0.369***
			(0.0256)			(0.0698)
Capital Control			0.651***			0.495**
			(0.0795)			(0.217)
Constant	1.723**	1.605***	1.531***	1.326***	-0.0425	3.179***
	(0.104)	(0.238)	(0.312)	(0.225)	(0.612)	(0.849)
GDP Growth and Interactive Term	-0.0170	-0.00826	-0.0307	0.647***	0.600***	0.457***
	(0.0400)	(0.0338)	(0.0324)	(0.0867)	(0.0869)	(0.0882)
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Number of Observations	395	395	340	391	391	340
Adj. R^2	0.020	0.318	0.436	0.146	0.160	0.266

Standard errors in parentheses. *, ** and *** indicate significance at the 10, 5 and 1% level, respectively.

Note: All flows are gross inflows as a percent of GDP. FDI refers foreign direct investment capital inflows. Debt refers both portfolio debt and other debt instruments inflows as in Balance of Payments. Annual data from 1980 to 2015. Countries included: Argentina, Brazil, Colombia, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Thailand, and Turkey. Source: World Bank and Alfaro et al. (2014).

Table 1 reports the regression results for FDI and debt inflows. For both these variables, the GDP growth rate is positively associated with inflows, suggesting that better economic conditions attract to foreign inflows in normal times. Moreover, this correlation is much more pronounced in the case of debt inflows. The interactive term, as measured by the product of GDP growth rate and a crisis dummy, shows the marginal effect of a crisis on the cyclicality of capital inflows. We notice that this term is positively associated with debt inflows, reflecting strongly procyclicality of external debt financing throughout the business cycle, contracting significantly during crises, and recovering during expansions. The behavior of FDI, however, is vastly different from debt around periods of crises. The coefficient of its interactive term is significantly negative, indicating that FDI is very resilient, and moves countercyclically during shocks that generate financial stress. These results are robust to controlling for the exchange rate regime, capital control policies and country fixed effects. The variability of US GDP contributes at best weakly to variations in capital inflows over the business cycle.

We also report the dynamic cyclical pattern of capital inflows during crises explicitly, which can be captured by the sum of the coefficients on the GDP growth rate and the interactive term. Notice that the sum of the two coefficients is slightly negative though insignificant. The implications of this result are discussed in more detail below.

During normal times, FDI inflows are procyclical. This is consistent with the results of authors of Ahmed and Zlate (2014), who have shown that domestic growth is a statistically and economically important determinant of private capital inflows. However, facing a financial shock, a deterioration in access to liquidity and a tightening of credit constraints may put a domestic firm in a more attractive position to foreigners. This is the fire-sale argument by Krugman (2000) and Aguiar and Gopinath (2005), which limits the negative impact of a growth slowdown on FDI inflows. In our analysis of emerging market economies during crises, this positive effect generated by financial shocks outweighs the negative impact of a growth slowdown, leading to a drop in the coefficient from positive (β_1) to negative ($\beta_1 + \beta_2$), implying a fall in the correlation between FDI and output between normal and crises times.

The above analysis shows the importance of financial shocks in driving cross-border capital flows in economic downturns and crises, and how it may help to stabilize inflows into emerging economies. With this in mind, in the next section, we build a small open economy model with financial frictions and MNC-owned firms, which takes into account the fire-sale channel created by a financial shock in an economic crisis.

3. Model

In this section, we construct a model of small open economy with financial frictions and FDI. The small open economy is populated by homogeneous households, capital producing firms and goods producing firms. The goods producing firms are either owned by domestic households or foreign MNCs. They produce a homogeneous good with capital and labor. Each firm borrows from an imperfect international financial market along the lines of Gertler and Karadi (2011) to finance its purchase of capital and accumulates net worth. When a firm enters it is owned by domestic investors. In each period there is a probability that the firm is acquired by a foreign MNC and becomes an MNC-owned firm. The associated capital inflows are FDI. MNC-owned firms are more productive and face looser borrowing constraints than domestic firms. Because of these, for a given net worth, an MNC-owned firm has a larger valuation than a domestically-owned firm. The system is subject to a productivity shock and a financial shock which affect domestic firms' ability to borrow in international financial markets.

3.1. Firms

There is a unit measure of firms $i \in [0, 1]$. Some are owned by domestic households and others by foreign MNCs. Within each firm type, firms are representative. To avoid confusion, we label a domestic firm with superscript d and an MNC-owned firm with superscript f. Firms produce with the following Cobb-Douglas production function:

$$y_{it}^s = A_t^s (k_{it-1}^s)^\alpha (l_{it}^s)^{1-\alpha}, \qquad s \in \{d, f\}, \tag{2}$$

where A_t^s denotes the productivity of type s firms, l_{it}^s denotes labor and k_{it-1}^s denotes the stock of capital for firm i.

Following Aguiar and Gopinath (2005) and Alquist et al. (2016), we assume that MNC-owned firms have higher productivity than domestically-owned firms. We assume $A_t^d = A_t$ and $A_t^f = \chi A_t$, where $\chi \geq 1$ is a reduced-form parameter to capture higher productivity in MNC-owned firms.

In period t, a firm i of type s has net worth n_{it}^s. It borrows b_{it}^{*s} from the international financial market at the world interest rate R_{t+1}^* to finance its purchase of capital $Q_t k_{it}^s$, where Q_t is the price of capital. The firm's balance sheet is given by:

$$n_{it}^s + b_{it}^{*s} = Q_t k_{it}^s. \tag{3}$$

After a firm produces, it sells undepreciated capital to capital producing firms and repays the loan with interest. The firm's net worth evolves as follows:

$$n_{it}^s = r_{kt}^s k_{it-1}^s + (1-\delta) Q_t k_{it-1}^s - R_t^* b_{it-1}^{*s}, \tag{4}$$

where the marginal product of capital of type-s firm r_{kt}^s is defined as $r_{kt}^s k_{it-1}^s \equiv \max_{l_{it}^s} \{y_{it}^s - w_t l_{it}^s\}$.[3] Labor is mobile across domestic and MNC-owned firms, so firms pay the same wage w_t. We also

[3]This means that:

$$\begin{aligned} l_{it}^s &= \left[\frac{(1-\alpha)A_t^s}{w_t}\right]^{\frac{1}{\alpha}} k_{it-1}^s, \\ r_{kt}^s k_{it-1}^s &= \alpha A_t^s \left[\frac{(1-\alpha)A_t^s}{w_t}\right]^{\frac{1-\alpha}{\alpha}} k_{it-1}^s. \end{aligned}$$

define the return on capital as:

$$R^s_{kt} \equiv \frac{r^s_{kt} + (1-\delta)Q_t}{Q_{t-1}}. \tag{5}$$

Given Cobb-Douglas production technology, the marginal product of capital is common across firms within each firm type. Since domestic firms are less productive than MNC-owned firms, but both types of firms face the same wage, domestically-owned firms have a lower return on capital.[4] The optimal choice of labor requires $w_t l^s_{it} = (1-\alpha)y^s_{it}$, and this implies that all firms have the same labor to output ratio.

We now describe the value of MNC-owned firms. After production takes place, in period $t+1$, there is an exogenous probability $(1-\kappa)$ that an MNC exits.[5] The MNC takes the net worth of the firm and leaves the small open economy. The firm faces financial frictions which makes its risk-adjusted return greater than the world interest rate, so it will keep accumulating assets until it leaves the industry. The firm maximizes its expected terminal wealth, given by:

$$V^f_{it} = \max_{k^f_{it}, b^{*f}_{it}} E_t\{\Lambda^*_{t,t+1}[(1-\kappa)n^f_{it+1} + \kappa V^f_{it+1}]\}, \tag{6}$$

where $\Lambda^*_{t,t+1} = 1/R^*_{t+1}$ denotes the stochastic discount factor of the foreign investors.

We construct the value function of a domestic firm in a similar way. Assume a domestic firm exits in the beginning of period $t+1$ with an exogenous probability σ. If it exits, the net worth is transferred back to households. If it does not exit, there is an exogenous probability Θ that it is acquired by a foreign MNC.[6] In this case the MNC acquires the domestic firm at a Nash-bargained price V^{nash}_{it+1} and this value is transferred back to domestic households who own equities of the domestic firm. With probability $\sigma(1-\Theta)$ the firm continues to operate as a domestic firm. The value of the firm is given by:

$$V^d_{it} = \max_{k^d_{it}, b^{*d}_{it}} E_t\{\Lambda_{t,t+1}[(1-\sigma)n^d_{it+1} + \sigma[\Theta V^{nash}_{it+1} + (1-\Theta)V^d_{it+1}]]\}, \tag{7}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor of domestic households.

Domestic firms and MNC-owned firms alike are subject to financial frictions. We assume financial frictions following Gertler and Karadi (2011). Specifically, after a firm borrows from the

[4] Easy to show that if $\chi > 1$, $r^f_{kt} = \chi^{\frac{1}{\alpha}} r^d_{kt} > r^d_{kt}$, and $R^f_{kt} > R^d_{kt}$.

[5] Following Carlstrom and Fuerst (1997), Bernanke et al. (1999) and Gertler and Karadi (2011), this assumption prevents firms from growing out of their financial constraints.

[6] In the benchmark model, we abstract from time-varying acquisition probability Θ. Aguiar and Gopinath (2005) and Alquist et al. (2016), however, find that the acquisition probability increases during financial crises. In Appendix D.2 we allow for time-varying acquisition probability and show that our main results are qualitatively unchanged.

international market, the firm manager has an option to divert a fraction of funds from the firm. If this happens, the firm will shut down. The firm manager will divert funds when the continuing value of the firm is less than the value of divertible capital. International lenders restrict their lending so that no firm managers divert funds in equilibrium. The incentive constraints for the lenders to MNC-owned firms and domestic firms are, respectively:

$$V_{it}^f \geq \theta^f Q_t k_{it}^f, \qquad V_{it}^d \geq \theta_t^d Q_t k_{it}^d. \tag{8}$$

where θ^f and θ_t^d represent the fraction of asset that each type of firm is able to divert.

We make two assumptions about the fraction of divertible assets. First, we assume that the fraction of divertible funds for the domestic firms, θ_t^d, follows an exogenous process, generating changes in firms' borrowing capacities. Jermann and Quadrini (2012) show that changes in credit conditions can strongly influence the dynamics of financial flows as well as the real business cycle, leading to economic downturns and financial crises. Therefore, we model a financial crisis as an exogenous positive shock in θ_t^d. This shock raises the fraction of divertible assets for domestically-owned firms, which tightens the financial constraint and reduces international lending to domestically-owned firms. Following Aguiar and Gopinath (2005), we assume that the fraction of funds divertible by MNCs, θ^f, is not affected. Because foreign MNCs are mainly from advanced economies, they can make their own line of financing available through other channels. Second, we assume that domestic firms face tighter incentive constraints than MNC-owned firms, reflecting their poorer access to international financial markets. This means that $\theta_t^d > \theta^f$.[7]

When an acquisition takes place, the MNC and domestic investors negotiate the acquisition price by splitting the surplus, $V_{it}^f - V_{it}^d$, via Nash bargaining. The match value V_{it}^{nash} is then given by:[8]

$$V_{it}^{nash} = \xi(V_{it}^f - V_{it}^d) + V_{it}^d, \tag{9}$$

where ξ is the domestic firm's relative bargaining power.

To sum up, domestic and MNC-owned firms maximize their value functions (7) and (6) respectively, subject to their balance sheets (3), evolution of net worth (4), the respective incentive constraints (8), and the Nash-bargaining condition (9), taking prices as given. We focus on the case in which both incentive constraints in (8) are binding. Because the value functions, balance sheets and the incentive constraints are all constant returns to scale, and because the Nash bargaining

[7] The calibration ensures that $\theta_t^d > \theta^f$ is satisfied for more than 99% of times.

[8] We will make explicit why a valuation gap exists later.

solution is also linear in the values of domestically-owned and MNC-owned firms, we conjecture that for each type of firm $s \in \{d, f\}$, the value of a firm is proportional to its net worth. Precisely, we define ψ_{it}^s as the marginal value per unit net worth:

$$\psi_{it}^s \equiv \frac{V_{it}^s}{n_{it}^s}, \qquad \text{for } \; s \in \{d, f\}, \tag{10}$$

and conjecture that $\psi_{it}^s = \psi_t^s$ for $s \in \{d, f\}$. Then, using (9), the Nash-bargained price per unit net worth can be expressed as a weighted average of domestic and MNC's valuation of a unit of net worth:

$$\frac{V_{it}^{nash}}{n_{it}^d} \equiv \psi_t^{nash} = \xi \psi_t^f + (1-\xi)\psi_t^d. \tag{11}$$

Binding incentive constraints (8) mean that all firms within each type s choose the same leverage $\phi_t^s \equiv \frac{Q_t k_{it}^s}{n_{it}^s}$, given by:

$$\phi_t^f = \theta^f \psi_t^f, \qquad \phi_t^d = \theta_t^d \psi_t^d. \tag{12}$$

By dividing the value functions by firms' net worth and substituting in the evolution of capital, one can show that the marginal values of net worth are given by:

$$\psi_t^s = \mu_t^s \phi_t^s + \nu_t^s, \qquad \text{for } \; s \in \{d, f\}, \tag{13}$$

where μ_t^f, ν_t^f, μ_t^d, and ν_t^d are given by:

$$\mu_t^f \equiv E_t\{\Lambda_{t,t+1}^* \Omega_{t+1}^* (R_{kt+1}^f - R_{t+1}^*)\}, \tag{14}$$

$$\nu_t^f \equiv E_t\{\Lambda_{t,t+1}^* \Omega_{t+1}^* R_{t+1}^*\}, \tag{15}$$

$$\mu_t^d \equiv E_t\{\Lambda_{t,t+1} \Omega_{t+1} (R_{kt+1}^d - R_{t+1}^*)\}, \tag{16}$$

$$\nu_t^d \equiv E_t\{\Lambda_{t,t+1} \Omega_{t+1} R_{t+1}^*\}, \tag{17}$$

and

$$\Omega_{t+1}^* \equiv (1-\kappa) + \kappa \psi_{t+1}^f, \tag{18}$$

$$\Omega_{t+1} \equiv (1-\sigma) + \sigma \Theta \psi_{t+1}^{nash} + \sigma(1-\Theta)\psi_{t+1}^d. \tag{19}$$

Here, Ω_{t+1}^* is the marginal value of net worth of MNC-owned firms in period $t+1$, which is the weighted average values of separating and continuing firms. Similarly, Ω_{t+1} is the marginal value of net worth of domestically-owned firms in period $t+1$. It is the weighted average of the values of exiting, matching with an MNC and continuing to operating as a domestically-owned firm. We define $\Lambda^*_{t,t+1}\Omega^*_{t+1}$ and $\Lambda_{t,t+1}\Omega_{t+1}$ as the 'augmented stochastic discount factor' for MNC-owned and domestic firms. (15) and (17) state that the marginal value of net worth, ν^s_t, is the expected product of the augmented stochastic discount factor and the world interest rate R^*_{t+1}. (14) and (16) state that the excess marginal value of capital, μ^s_t, is the expected product of the augmented stochastic discount factor and the excess return $(R^s_{kt+1} - R^*_{t+1})$.

Finally, the incentive constraints can be rearranged to solve for the leverages:

$$\phi^f_t = \frac{\nu^f_t}{\theta^f - \mu^f_t}, \qquad \phi^d_t = \frac{\nu^d_t}{\theta^d_t - \mu^d_t}. \tag{20}$$

To make sure the constraints (8) are binding, we require that (1) $\theta^f > \mu^f_t$, (2) $\theta^d_t > \mu^d_t$, (3) $\psi^d_t > 1$, and (4) $\psi^f_t > 1$. The first two inequalities ensure that at high enough leverage, firms have an incentive to divert funds. The last two inequalities ensure that it is always profitable for firms to continue to operate. We check that these constraints are satisfied around the non-stochastic steady state when we solve the model later on.

To understand why domestic investors and foreign MNC value domestic firms differently, we combine (13), (14), (15), (16) and (17) to get:

$$\psi^f_t = E_t(\Lambda^*_{t,t+1}\Omega^*_{t+1}[(R^f_{kt+1} - R^*_{t+1})\phi^f_{t+1} + R^*_{t+1}]), \tag{21}$$

$$\psi^d_t = E_t(\Lambda_{t,t+1}\Omega_{t+1}[(R^d_{kt+1} - R^*_{t+1})\phi^d_{t+1} + R^*_{t+1}]). \tag{22}$$

These value functions are different in four aspects. First, MNC brings about technology spillovers, so $R^f_{kt} > R^d_{kt}$. Second, an MNC-owned firm faces looser financial constraints than domestic firms $\theta^f < \theta^d_t$. So for a given amount of net worth, an MNC-owned firm can borrow more and have higher leverage, *i.e.*, $\phi^f_{t+1} > \phi^d_{t+1}$. Third, domestic households do not have access to international financial markets, and they discount more heavily than foreign MNCs. These three effects increase an MNC's valuation of a domestic firm relative to domestic investors' valuation. Finally, the survival rate of MNC firms, σ may be different from that of the domestic firms κ. The calibration of the model is such that ψ^f_t is bigger than ψ_t around the steady state, so the foreign MNC is always willing to buy a domestic firm.

3.2. Capital goods producers

At the end of each period, domestically-owned and MNC-owned firms alike sell undepreciated capital to competitive capital goods producers owned by domestic households. A representative capital good producer buys previously installed capital and combines with investment good I_t from final goods producers to produce new capital. Newly produced capital is sold back to the firms within the same period. Production of new capital is subject to convex investment adjustment costs. The evolution of capital is given by:

$$K_t = (1-\delta)K_{t-1} + (1 - Adj_t)I_t, \tag{23}$$

where $Adj_t = \frac{\Psi^I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ are investment adjustment costs. Capital goods producers maximize discounted sum of expected future profits, $E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\Pi^K_{t+s}$, where $\Pi^K_t = Q_t[K_t - (1-\delta)K_{t-1}] - I_t$. The first order condition for the optimal investment choice is:

$$1 = Q_t\left[1 - Adj_t - \Psi^I \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + E_t\left[\Lambda_{t,t+1}Q_{t+1}\Psi^I\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right]. \tag{24}$$

3.3. Domestic Households

Infinite-lived representative households in the small open economy derive utility from consumption and disutility from supplying labor. The representative households' preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln\left(C_t - \Psi^L \frac{L_t^{1+\varphi}}{1+\varphi}\right). \tag{25}$$

We use Greenwood et al. (1988) (GHH) preferences. It is well-known that the utility can generate a labor supply schedule that only depends on the real wage. Moreover, Correia et al. (1995) and Raffo (2008) show that GHH preferences are better suited to match the second moments of open economies.

In each period, a representative household receives wage income, returns from holding domestic bonds D^d_t and equities of domestic firms s_{it}, and profits from capital producing firms Π^K_t. The household consumes, adjusts its asset portfolio, and pays startup funds to new domestically-owned firms, denoted as tr_t. To sum up, a representative household faces the following budget constraint:

$$w_t L_t + R^d_t D^d_{t-1} + \int_i s_{it-1}[(1-\sigma)n^d_{it} + \sigma\Theta V^{nash}_{it} + \sigma(1-\Theta)V^d_{it}]di + \Pi^k_t = C_t + \int_i s_{it}V^d_{it}di + D^d_t + tr_t. \tag{26}$$

The intratemporal labor supply conditions are given by the following:

$$w_t = \Psi^L L_t^{\varphi}. \tag{27}$$

The following consumption Euler equation helps us pin down the domestic interest rate:

$$1 = E_t(\Lambda_{t,t+1} R_{t+1}^d), \tag{28}$$

where the stochastic discount factor is given by $\Lambda_{t-1,t} = \beta U_{C,t}/U_{C,t-1}$. The optimal choice for equity turns out to be a restatement of the solved-out value function of domestic firms.[9] Clearly, this is just Modigliani and Miller (1958) theorem at work.

3.4. Aggregation and market clearing

Since each type of firms have the same capital to labor ratio and leverage ratio, we only need to keep track of the sector level quantities. For $Z \in \{Y, K, L, N, B^*\}$, we define $Z_t^d \equiv \int_i z_{it}^d di$, $Z_t^f \equiv \int_i z_{it}^f di$, and we also define economy-wide variables such that $Z_t \equiv Z_t^f + Z_t^d$.

The aggregate balance sheets of the domestically-owned firms and MNC-owned firms are given by:

$$B_t^{*s} \equiv Q_t K_t^s - N_t^s, \qquad \text{for } \ s \in \{d, f\}. \tag{29}$$

Next, we derive the law of motion of the net worth of MNC-owned and domestically-owned firms. In each period, a fraction $(1 - \sigma)$ of domestically-owned firms exits the market. To make the number of firms in the economy constant, we assume that an equal measure of new domestic firms enters, with start-up funds transferred from domestic households. In sum, the net worth of domestic firms evolves as follows:

$$N_t^d = \sigma(1 - \Theta)[(R_{kt} - R_t^*)\phi_{t-1}^d + R_t^*]N_{t-1}^d + \omega Q_t K_{t-1}^d, \tag{30}$$

where $\omega Q_t K_{t-1}^d$ is the start-up fund.

[9]The Euler equation for equity is given by:

$$V_{it}^d = E_t \left(\Lambda_{t,t+1}[(1-\sigma)n_{it+1}^d + \sigma\Theta V_{it+1}^{nash} + \sigma(1-\Theta)V_{it+1}^d] \right).$$

We divide the above equation by n_{it}^d, and get $\psi_{it}^d = \psi_t^d = E_t(\Lambda_{t,t+1}\Omega_{t+1}[(R_{kt+1}^d - R_{t+1}^*)\phi_t^d + R_{t+1}^*]) = \mu_t^d \phi_t^d + \nu_t^d$, which is the same as (13).

For MNC-owned firms, matches dissolve with an exogenous separation rate $(1-\kappa)$. When a multinational separates from a local firm, it takes the net worth. The net worth of firms owned by foreign MNCs evolves as follow:

$$N_t^f = \kappa[(R_{kt} - R_t^*)\phi_{t-1}^f + R_t^*]N_{t-1}^f + \sigma\Theta[(R_{kt} - R_t^*)\phi_{t-1}^d + R_t^*]N_{t-1}^d. \tag{31}$$

The first term on the right hand side refers to MNC-owned firms that survive after period $t-1$ and the second term refers to the firms newly acquired by MNCs in period t.

The resource constraint in this economy is given by the following balance of payment equation:[10]

$$\underbrace{Y_t - C_t - I_t}_{\text{net exports}} = \underbrace{\underbrace{(1-\kappa)[R_{kt}Q_{t-1}K_{t-1}^f - R_t^* B_{t-1}^{*f}]}_{\text{FDI outflows}} - \underbrace{\sigma\Theta V_t^{nash}}_{\text{FDI inflows}}}_{\text{equity financing}} + \underbrace{R_t^* B_{t-1}^* - B_t^*}_{\text{debt financing}}. \tag{32}$$

The left hand side of this equation is net exports; the right hand side is the capital account, which comprises FDI (equity) financing and debt financing. Finally, asset markets clear, which means that $D_t^d = 0$ and $s_{it} = 1$, for all i.

3.5. Shock processes

To make our model parsimonious, we assume only two exogenous shocks in this system, namely a TFP shock and a shock to the financial constraint facing domestic firms while setting the world interest rate R^* equal to a constant.[11] We assume that these shocks follow exogenous AR(1) processes as follows:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{At}, \qquad \epsilon_{At} \sim N(0, \sigma_A^2), \tag{33}$$

$$\ln \theta_t^d = (1-\rho_\theta)\ln\bar{\theta}^d + \rho_\theta \ln\theta_{t-1}^d + \epsilon_{\theta t}, \qquad \epsilon_{\theta t} \sim N(0, \sigma_\theta^2), \tag{34}$$

where we use an upper bar to denote the steady state of a variable. The innovations of all shocks are assumed to be *i.i.d*, uncorrelated over time and with each other.

[10] Appendix B shows the derivation of the balance of payments equation.

[11] We are aware that the world interest rate shock is another driver of business cycles in emerging economies (see for example Uríbe and Yue (2006) and Neumeyer and Perri (2005)). Appendix D.1 shows that the world interest rate affects FDI and external debt in a way similar to a productivity shock, so it does not help account the dynamic pattern of FDI during crises.

Table 2
Calibrated parameters.

Parameter	Value	Meaning
β	0.985	Subjective discount factor
α	0.33	Capital share in production
δ	0.025	Capital depreciation rate
Ψ^L	5	Labor disutility
φ	1	Inverse of Frisch labor elasticity
Ψ^I	2.5	Convexity of investment adjustment costs
R^*	$1.04^{1/4}$	World interest rate
κ	0.921	Domestic firm survival rate
σ	0.96	MNC-owned firm survival rate
ξ	0.3	Domestic firm bargaining weight
χ	1.1^{α}	Technology spillovers by MNC
$\bar{\theta}^d$	0.71	Fraction of divertible assets, domestic firms
θ^f	0.56	Fraction of divertible assets, MNC-owned firms
ω	0.0123	Start-up funds for domestic firms
Θ	0.002	MNC acquisition probability
ρ_A	0.95	Persistence of productivity shock
ρ_θ	0.97	Persistence of financial shock
σ_A	0.0037	Std. dev of productivity shock innovation
σ_θ	0.025	Std. dev of financial shock innovation

4. Calibration

In the following we solve and simulate the model numerically. The model is solved by using log-linear approximation of the system around its non-stochastic steady state. This section discusses our calibrations to match the model with emerging economies' business cycles.

Each period is a quarter. Parameters in production and household sectors are relatively standard in the macroeconomic literature. These are given in Table 2. We set $\beta = 0.985$, which generates a steady state annualized interest rate around 6%. We set $\Psi^L = 5$, so that households devote 37 percent of their time to work. The parameter that governs the Frisch elasticity of labor supply is set to $\phi = 1$. For production, the capital share is set to $\alpha = 0.33$, and the depreciation rate to $\delta = 0.025$. The curvature of investment adjustment costs Ψ^I is set to 2.5. Lastly, we set the world interest rate to $R^* = 1.04^{1/4}$.

We calibrate non-standard parameters in the model as follows. We set $\sigma = 0.96$ which implies that a domestic firm is expected to survive for about 6 years.[12] We follow Smith and Valderrama (2009) to set the MNC-owned firm survival probability to $\kappa = 0.921$. That MNC-owned firms are more likely to quit is consistent with empirical evidence (Ibarra-Caton (2012), Ferragina et al. (2009), and Aguiar and Gopinath (2005)). We set moderate technology spillovers by MNC to

[12] Morris (2009) estimates that US firms have average life expectancies of 7 to 11 years.

$\chi = 1.1^{\alpha}$, so that the return on capital by MNC-owned firms is higher than that of domestically-owned firms. We set the relative bargaining weight of domestic firm to $\xi = 0.3$.

We calibrate the credit contract parameters to match four steady-state targets. First, we set the steady-state leverage of domestic firms to $\phi^d = 1.7$. Second, the steady-state external finance premium for domestic firm is set to $R_k^d/R^* = 1.007$. These two values come from the emerging market dataset in Fernández and Gulan (2015). Third, we set the stock of FDI liability to GDP ratio to $V^f/(4Y) = 9\%$. This is the average of the stock of FDI liability to GDP ratio in our sample countries in 1980-2007, according to the External Wealth of Nations dataset (Lane and Milesi-Ferretti (2007)).[13] Fourth, the fraction of capital owned by domestic firms is set to $K^d/K = 0.92$. This fraction is consistent with Smith and Valderrama (2009) and Mendoza and Smith (2006). These targets identify θ^f, $\bar{\theta}^d$, Θ, and the steady-state MNC-owned firm leverage ϕ^f uniquely. See Appendix C for details.

These steady state conditions imply $\theta^f = 0.56$ and $\bar{\theta}^d = 0.71$. Importantly, $\theta^f < \bar{\theta}^d$, which implies that domestic firms face tighter financial constraints than MNC-owned firms. The steady-state leverage ratio of MNC-owned firms is $\bar{\phi}^f = 3.1$, which is larger than that of domestic owned firms, reflecting the fact that MNC-owned firms face looser financial conditions. In the steady state, $\bar{\psi}^f = 1.73 > \bar{\psi} = 1.20$, which suggests that foreign multinationals indeed have an incentive to acquire domestic firms. Furthermore, we obtain $\Theta = 0.002$, which implies that 0.8% of domestic firms gets acquired in a year, or an FDI to output ratio of 1.1%. The start-up fund parameter ω is set to 0.0123.

We calibrate productivity shocks and financial shocks following Kalemli-Ozcan et al. (2013)'s strategy. Precisely, we assume that the financial shock is turned off in non-crisis periods and is turned on during crises. As good data for the Solow residual for emerging markets is not available, we rely on output data in crisis and non-crises time to back out the remaining shock processes. We follow Fernandez-Villaverde et al. (2011) and Mendoza (1991) to set $\rho_A = 0.95$. We use HP-filtered (Hodrick and Prescott (1997)) quarterly log-output data in 1990q1-2012q3 for all emerging economies in our sample, and split the sample according to crisis and non-crisis periods. We find that the output volatility is around 2.5 % in non-crisis periods, and 3.3 % in crisis periods. These numbers accord well with the existing literature on emerging economies' business cycles, for

[13] We find from the same dataset by Lane and Milesi-Ferretti (2007) that the external debt liability to annual GDP ratio in Korea, Philippines and Thailand are 1980-2007 are 32%, 64% and 44% respectively, whereas the same steady-state ratio implied by our baseline model is 85%. One key reason is that firms in our model cannot borrow domestically. Later we will discuss an extended model which allows for domestic loans together with external debt and in that model we can match the observed external borrowing to GDP ratio.

instance, Aguiar and Gopinath (2007). We then calibrate the standard deviation of productivity innovation to $\sigma_A = 0.0037$ and the standard deviation of financial shock innovation to $\sigma_\theta = 0.025$.[14] We set the persistence of financial shocks to $\rho_\theta = 0.97$. With this calibration, when both shocks are turned on (crisis periods), the volatility of output is 3.0%. With only productivity shocks turned on (normal times), the volatility of output is 2.5 %.[15]

5. Model properties

This section discusses the simulation results of our model. We proceed as follows. First we report the impulse responses when the model is hit by productivity shocks and financial shocks respectively. Next, we compute simulated moments and compare them with the data. We show that the model-generated moments match the stylized facts fairly well. Last, we conduct comparative static analysis to further explore the role of financial frictions in generating the cyclical pattern of FDI in the model.

5.1. Impulse responses

Fig. 2 and 3 show the responses of key macro and financial variables to one standard deviation adverse productivity and financial shocks respectively. In both cases, the economy starts from the steady state and is hit by one of the shocks at time 0. All variables are expressed in their percentage deviations from the steady state.

In Fig. 2, a negative productivity shock leads to a sharp fall in the realized return on capital for both domestically-owned firms and MNC-owned firms. The net worth of both sectors falls, which further depresses investment demand via a financial accelerator mechanism as in Gertler and Karadi (2011). The fall in net worth reduces equity prices V_t^d for domestic firms initially. With impaired balance sheets, total international lending to the small open economy B_t^* falls.

The total value of FDI inflows ($\mathrm{FDI_t} = \sigma\Theta\psi_t^{nash}N_t^d$) is affected by a volume effect and price effect, which work in opposite directions. The volume effect refers to the fact that acquired firms have a smaller net worth on average after a negative shock. The price effect refers to a rise in ψ_t^{nash}, the Nash-bargained acquisition value per unit of net worth. With a sharp fall in investment

[14]The standard deviation for the innovation process of TFP is smaller than what is found in other work, such as Fernandez-Villaverde et al. (2011) and Neumeyer and Perri (2005) for two reasons. First, financial frictions in our model amplifies shocks. Second, we exclude the crisis periods when we calibrate the productivity shock and attribute increased volatility during crises to financial shocks.

[15]To match the volatility of GDP exactly would require an unreasonably volatile financial shock, in which case the incentive constraints are violated too often.

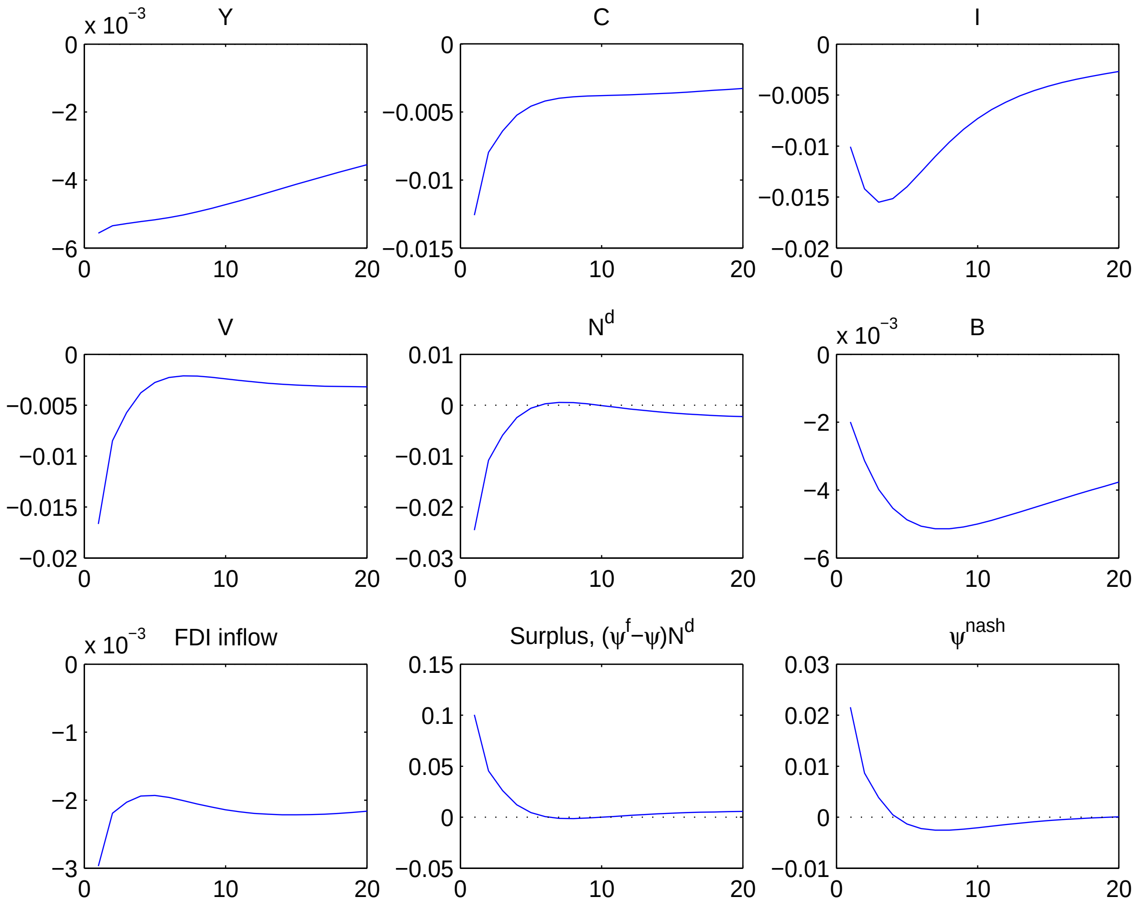

Fig. 2. Impulse response to a negative productivity shock. *Note*: The impulse response functions measure the response to a one standard deviation negative shock to the innovations of TFP as the percent deviation from the steady state. The series are simulated based on the benchmark calibration.

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the expected future return on capital increases, which raises the expected return on capital and value per unit of net worth for both types of firm. This effect is more pronounced for MNC-owned firms because they face looser financial constraints and are able to borrow with higher leverage. As a result, the valuation gap per unit of net worth ($\psi_t^f - \psi_t^d$) widens. As ψ^f rises by more than ψ^d, the Nash-bargained acquisition price ψ_t^{nash} also rises by more than ψ^d. In our calibrated model, the volume effect dominates when the system is hit by a productivity shock and FDI inflows fall. Therefore, there is positive comovement between international debt and FDI inflow.

Fig. 3 shows the impulse responses to an adverse financial shock θ_t. Recall that a negative financial shock is an unexpected rise in the fraction of divertsifiable assets in the domestic firm sector. As MNC-owned firms have access to international financial markets via MNCs, the fraction of divertsifiable assets in MNC-owned firm sector is not affected.

In response to a tightening of financial constraints, domestic firms borrow less in international financial markets, so B_t^* falls. A fall in leverage in domestic firms reduces investment demand and decreases the price of capital Q_t, which reduces the value of capital in both types of firms. As a result, net worth in both sectors drops and the expected future return on capital increases. Contrary to a productivity shock, a financial shock only directly affects domestic firms which tightens their financial constraints but that is not the case for MNC-owned firms, so an increase in leverage happens disproportionately in MNC-owned firms. The valuation gap per unit of net worth ($\psi_t^f - \psi_t^d$) between MNC-owned firms and domestic firms increases sharply, and so is the acquisition price per unit of net worth ψ_t^{nash}. With a strong price effect, FDI inflows turn positive after a financial shock, leading to positive comovement between international debt and FDI inflows.

5.2. Simulated moments

Next we compare simulated moments in our model with emerging market economy data in Table 3. The first two columns show the key empirical moments in emerging economies. Since the model has quarterly frequency, we re-calculate the empirical second moments with quarterly data in our sample emerging economies.[16] The finding is similar to what we obtained in Section 2 using annual data. In particular, the first two rows show that, FDI and debt are both procyclical in normal times, but the correlation between FDI in output falls substantially during crises and becomes slightly negative.

[16]Countries included: Argentina, Brazil, Colombia, Korea, Malaysia, Mexico, Philippines, Thailand, and Turkey. Due to data availability, we exclude Indonesia and Peru.

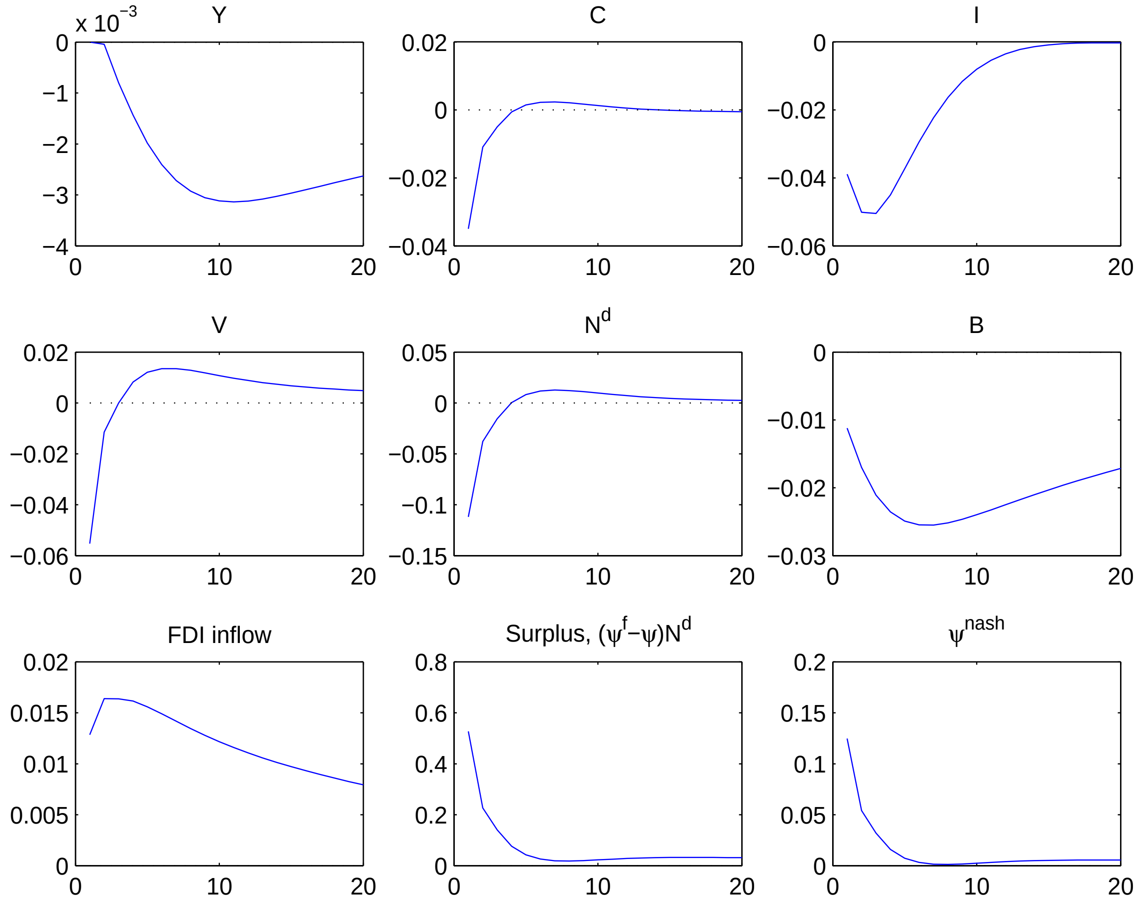

Fig. 3. Impulse response to an adverse financial shock. *Note*: The impulse response functions measure the response to a one standard deviation positive shock to the innovations of the credit constraint as the percent deviation from the steady state. The series are simulated based on the benchmark calibration.

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Table 3
Business cycle statistics.

	Emerging economies		Benchmark model		Model with exchange rate	
	(1)	(2)	(3)	(4)	(5)	(6)
	Normal	Crisis	Normal	Crisis	Normal	Crisis
$\rho(\mathrm{Y,FDI})$	0.22	-0.10	0.94	-0.21	0.95	-0.15
$\rho(\mathrm{Y,Debt})$	0.22	0.20	0.98	0.66	0.71	0.45
$\rho(\mathrm{I,FDI})$	0.11	-0.10	0.64	-0.60	0.75	-0.67
$\rho(\mathrm{I,Debt})$	0.21	0.23	0.78	0.57	0.68	0.32
σ_Y (%)	2.54	3.27	2.53	2.98	2.49	2.95
σ_C/σ_Y	1.06	1.37	1.07	1.55	0.90	0.85
σ_I/σ_Y	3.76	4.69	1.63	3.81	1.56	3.22

Note: Moments of emerging economies are computed by using quarterly data from 1990Q1 to 2012Q3. Source: IFS. The numbers from the model are the averages of 100 series of 2100 periods simulated based on the benchmark calibration. σ_Y denotes the standard deviation of GDP (in percent). σ_i/σ_Y represents the standard deviation relative to that of GDP. $\rho(Y,i)$ is the correlation with GDP. $\rho(I,i)$ is the correlation with investment.

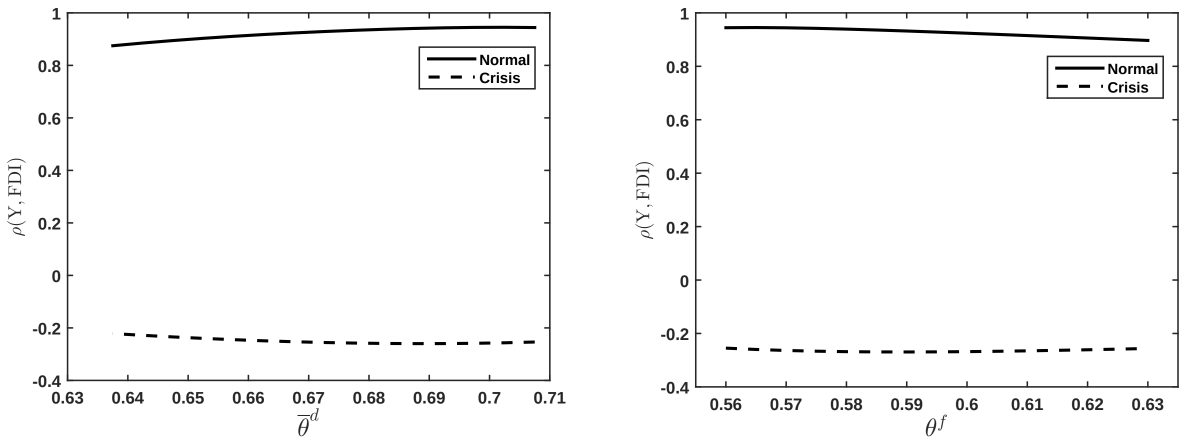
Column 3 and 4 of Table 3 report the simulated moments of the model with productivity shocks in normal times and with both productivity and financial shocks in crises times. The first row shows that the model generates a positive correlation between output and FDI in normal times. During crises, however, the correlation drops to -0.21. The second row shows that the correlation between output and debt is positive throughout the business cycle. These patterns are broadly consistent with our empirical findings. Row 3 and 4 of Table 3 report the empirical and simulated correlations with investment. Both the data and model show a significant reduction in correlation between investment and FDI during crises.

We also report the standard deviation of consumption and investment. Our benchmark model generates a standard deviation of consumption slightly larger than that of output in normal times, consistent with the observed data. Consumption is too volatile during crises in our model. One reason is that we assume firms can only borrow from international investors. The way we calibrate the model to match the leverage ratio of domestic firms makes international borrowing artificially large.[17] When there is a crisis, international lenders cut back their lending, and so the current account needs to have a large surplus. For given output, both consumption and investment have to fall sharply to satisfy the balance of payments identity. Our model does not generate enough investment volatility in normal times. We show in Appendix D.1 that when world interest rate shocks are present, the model can generate σ_I/σ_Y close to the data.

5.3. Comparative statics

Finally, we study to what extent differential access to international financial markets by domestic and MNC-owned firms affects the correlation between FDI and output. The key to our model's

[17] In the steady state, external debt is 86% GDP, whereas in the data it is about 50%.



(a) Varying $\bar{\theta}^d$ **(b)** Varying θ^f

Fig. 4. Financial frictions and correlation. *Note*: Comparative static analysis of changing $\bar{\theta}^d$ and θ^f in normal and crises times. $\rho(\mathrm{Y}, \mathrm{FDI})$ denotes the correlation between output and FDI.
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ability to capture the dynamics of capital inflows in different phases of the business cycle is the relative importance of the financial shock, which itself depends on the uneven degrees of financial frictions. That is our results depend on the relative difference in credit constraints faced by the domestic and MNC-owned firms. In turn, this difference depends on the fraction of divertible assets faced by each type of firm, $\bar{\theta}^d$ and θ^f. We conduct comparative static exercises of the model in normal and crises times separately, and we vary the steady-state fraction of divertible assets of domestic firms $\bar{\theta}^d$ and the fraction of divertible assets of MNC-owned firms θ^f. Figure 4 reports the results. In panel (a), an improvement to domestic firm's access to the asset market (a decrease in $\bar{\theta}^d$) shrinks the difference in the tightness of the credit constraint $\bar{\theta}^d - \theta^f$, decreases the valuation difference between domestic households and foreign investors. As a result, the importance of financial frictions and financial shocks fall. Reflected on the second moments, the difference in correlation $\rho(\mathrm{Y}, \mathrm{FDI})$ between normal times and crises times becomes smaller. Panel (b) conducts a similar exercise by keeping $\bar{\theta}^d$ constant and raising θ^f. Again this makes the two types of firms more similar in their ability to access the credit market. Consequently, the difference in $\rho(\mathrm{Y}, \mathrm{FDI})$ in normal times and crises times falls.

6. Real exchange rate and endogenous portfolio choice

The discussion so far abstracts from the real exchange rate channel. A sharp exchange rate depreciation during a financial crisis makes physical assets in the small open economy much cheaper from a foreign investors' perspective. Erel et al. (2012) provide empirical evidence that the exchange rate of the host country tends to depreciate before an acquisition. In this extension, we incorporate endogenous real exchange rate adjustments into our framework. By doing so, we also allow firms to borrow both domestically and in international financial markets. We model a portfolio choice problem by firms following Aoki et al. (2016). In equilibrium, due to financial frictions, firms borrow both domestically and internationally. In what follows, we sketch the structure of the model and leave the details of the model in Appendix D.3.

We start by describing the firms. There are still domestic firms (with superscript d) and MNC-owned firms (with superscript f). Firms use not only capital and labor, but also imported goods to produce with Cobb-Douglas technologies:

$$y_{it}^s = A_t^s (k_{it-1}^s)^{\alpha_K} (l_{it}^s)^{\alpha_L} (m_{it}^s)^{\alpha_M}, \tag{35}$$

where m_{it}^s denotes imports by firm i of type $s \in \{d, f\}$.

A firm has net worth n_{it}^s and can now obtain domestic borrowing b_{it}^s and foreign borrowing $S_t b_{it}^{*s}$ to purchase capital $Q_t k_{it}^s$. S_t denotes the real exchange rate. The firm's balance sheet is:

$$n_{it}^s + b_{it}^s + S_t b_{it}^{*s} = Q_t k_{it}^s. \tag{36}$$

We introduce financial frictions following Aoki et al. (2016). We assume a firm's ability to divert funds depends on the sources and uses of funds. Specifically, we assume $V_{it}^s \geq \theta_t^s (1 + 0.5\gamma^s (x_{it}^s)^2) Q_t k_{it}^s$, where $x_{it}^s \equiv \frac{S_t b_{it}^{*s}}{Q_t k_{it}^s}$ is the fraction of international borrowing. If $x_{it}^s = 0$, then θ_t^s is the fraction of divertible fund when a firm only borrows in the domestic financial market. A positive γ^s means that the firm can divert a larger fraction of assets when it borrows in international financial markets x_{it}^s.

When the exchange rate is included, foreign investors take into account expected exchange rate changes when they evaluate the value of the firms in the small open economy. The value of an MNC-owned firm is now given by:

$$V_{it}^f = \max E_t \left\{ \Lambda_{t,t+1}^* \frac{S_t}{S_{t+1}} \left[(1-\kappa) n_{it+1}^f + \kappa V_{it+1}^f \right] \right\} \tag{37}$$

where $\Lambda_{t,t+1}^* = 1/R_{t+1}^*$ denotes the stochastic discount factor of foreign investors.[18]

[18] Suppose foreign investors can save by buying foreign risk-free bonds which pays a return of R_t^*, or by investing in MNC-owned firms in the small open economy. The optimal choice of foreign bonds satisfies $E_t(\Lambda_{t,t+1}^* R_{t+1}^*) = 1$. Our approximation is correct up to the first-order approximation. The optimal choice of stocks of MNC-owned domestic

Domestic firms have production function analogous to that of MNC-owned firms. We continue to assume that domestic firms face tighter incentive constraints than MNC-owned firms, which means that $\theta_t^d > \theta_t^f \equiv \theta^f$. Matching between domestic firms and MNCs lead to acquisition of domestic firms, and the acquisition value is given by (9). If the financial constraints are always binding, firms' values are given by:

$$\psi_t^s = \mu_t^s \phi_t^s + \eta_t^s \phi_t^s x_t^s + \nu_t^s, \qquad \text{for } \; s \in \{d, f\}.$$

The linearity of the value functions are still preserved, and so all firms within each type choose the same leverage ϕ_t^s and share of foreign borrowing x_t^s, given by:

$$\phi_t^s = \frac{\nu_t^s}{\theta_t^s \left(1 + \frac{\gamma^s}{2}(x_t^s)^2\right) - \mu_t^s - \eta_t^s x_t^s}, \tag{38}$$

$$x_t^s = \frac{\mu_t^s}{\eta_t^s}\left(-1 + \sqrt{1 + \frac{2}{\gamma^s}\left(\frac{\eta_t^s}{\mu_t^s}\right)^2}\right), \qquad \text{for } \; s \in \{d, f\}. \tag{39}$$

As before, the variables ν_t^s and μ_t^s denote the marginal value of net worth and excess marginal value of capital for type-s firms respectively. The variable η_t^s denotes the cost advantage of foreign borrowing relative to domestic borrowing for type-s firm. Equation (39) states that, *ceteris paribus*, a rise in η_t^s increases the share of foreign borrowing for firm type-s. The expressions for ν_t^s, μ_t^s, and η_t^s are given in Appendix D.3.

Households and capital goods producers face identical problems as in the benchmark model.

The external account of the small open economy is given by:

$$export_t - S_t M_t = (1-\kappa)(R_{kt}^f Q_{t-1} K_{t-1}^f - R_t^d B_{t-1}^f - R_t^* S_t B_{t-1}^{*f}) - \sigma\Theta V_t^{nash} + S_t(R_t^* B_{t-1}^* - B_t^*), \tag{40}$$

where foreign demand of domestic good is assumed to follow $export_t = S_t^{\varsigma} \bar{export}$, where ς is the exchange rate elasticity of exports. In equilibrium, domestic households save and finance all domestic loans.

We calibrate additional parameters as follows. We choose $\gamma^d = 5$ and $\gamma^f = 1.5$, reflecting the fact that MNC-owned firms can better manage their foreign debts. The import share in production α_M is set to 0.13 so that steady-state import is 15% of GDP. We set $\varsigma = 1.5$. In the steady state,

firms satisfies

$$E_t \left[\Lambda_{t,t+1}^* \left(R_{t+1}^* - \frac{S_t}{S_{t+1}} \frac{[(1-\kappa)n_{it+1}^f + \kappa V_{it+1}^f]}{V_{it}^f} \right) \right] = 0.$$

$\bar{x}^d = 0.24\%$ of domestic firms' debt is borrowed in international financial markets, whereas the proportion of foreign borrowing is $\bar{x}^f = 35\%$. Furthermore, the model implies a steady-state external debt to GDP ratio of 55%, consistent with the observed data.

In effect the real exchange rate appears in three places in the model: (1) imports and exports; (2) foreign currency loans in domestic and MNC-owned firms; and (3) foreign investors valuation of MNC-owned firms. We consider how real exchange rate adjustments affect $\rho(\mathrm{Y}, \mathrm{FDI})$ through these channels.

In response to a financial shock the real exchange rate depreciates, so imports become more expensive. When the import share of production is large, a real exchange rate depreciation hurts firms more. Their net worth drops by more initially. As a result, the average firm acquired by MNCs is smaller, and FDI inflows are smaller. In other words, when α_M is larger, $\rho(\mathrm{Y}, \mathrm{FDI})$ is less negative.

Exchange adjustments also affect the portfolio choice of firms. As a financial shock hits, the real exchange rate depreciates immediately and there is an appreciation expectation along the adjustment path. This makes borrowing in international financial markets more attractive and triggers firms to adjust their portfolio towards foreign borrowing. Such portfolio adjustment reduces the current account surplus and the fall in consumption and investment. This results in a smaller fall in Q and in firms' net worth. With a smaller volume effect, portfolio adjustment tends to make Y and FDI more negatively correlated.

Finally, as shown in (37), expected real exchange rate appreciation also increases foreign investors valuation gap directly, thus increasing the attractiveness of acquisition.

Column (5) and (6) of Table 3 compare the second moments generated by this model with the data and the benchmark model. The second moments generated by this model are quite similar to the benchmark model. Importantly, the extended model generates a substantial fall in $\rho(\mathrm{Y}, \mathrm{FDI})$ from 0.95 to -0.15 from normal to crises times. This result is consistent with our stylized fact that FDI is resilient during crises; it moves countercyclically so its correlation with GDP drops sharply. We conclude that our main results are qualitatively unchanged in this richer model with an embedded real exchange rate channel and firms' endogenous portfolio choices.

7. Conclusion

In this paper we study the cyclical behavior of FDI and external debt inflows to emerging economies. We show that FDI in emerging economies moves procyclically in normal times but not so much during crises. We develop a theoretical framework featuring financial frictions and financial shocks to analyze the dynamic pattern of FDI and debt financing in emerging economies. Our model successfully produces positive correlations between FDI and external debt in normal times and negative correlations during crises. We embed a credit constraint in a small open economy set up. The existence of an uneven degree of financial frictions facing domestically-owned and MNC-owned firms generates flows of direct investment from MNCs to domestic firms in emerging economies. Facing a negative financial shock, the wedge between the valuations of MNC-owned and domestically-owned firms increases leading to foreign direct investment by MNCs into emerging economies. The deterioration in financial conditions also reduces the borrowing ability of domestic firms and, as a result, debt falls. Therefore, the model successfully accounts for both a significant decline in the external debt position of emerging economies during economic crises, and the relative stability of FDI.

The findings of this paper have important policy implications. The liberalization of financial markets has resulted in large capital inflows to emerging economies. However, this raises the risk of possible destabilizing macroeconomic effects created by short-term debt inflows. Our results show that unlike FDI, debt inflows are strongly cyclical and have an amplification effect on the economic cycle during crises. With an imperfect financial market and excessive leveraging built up in good times, the danger of sudden stops in debt inflows may cause economic disruption. Therefore, from the perspective of stabilizing economic fluctuations, countercyclical financing like FDI offers a more promising avenue to development in emerging economies.

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Appendix A. Full system

Firms acquired by foreign MNCs:

$$Y_t^f = \chi A_t (K_{t-1}^f)^\alpha (L_t^f)^{1-\alpha} \tag{A.1}$$

$$w_t L_t^f = (1-\alpha) Y_t^f \tag{A.2}$$

$$R_{kt}^f = \frac{\alpha \frac{Y_t^f}{K_{t-1}^f} + (1-\delta) Q_t}{Q_{t-1}} \tag{A.3}$$

$$\phi_t^f N_t^f = Q_t K_t^f \tag{A.4}$$

$$\phi_t^f = \frac{\nu_t^f}{\theta^f - \mu_t^f} \tag{A.5}$$

$$\mu_t^f = E_t\{\Lambda_{t,t+1}^* [(1-\kappa) + \kappa \theta^f \phi_{t+1}^f](R_{kt+1}^f - R_{t+1}^*)\} \tag{A.6}$$

$$\nu_t^f = E_t\{\Lambda_{t,t+1}^* [(1-\kappa) + \kappa \theta^f \phi_{t+1}^f] R_{t+1}^*\} \tag{A.7}$$

Domestic firms:

$$Y_t^d = A_t (K_{t-1}^d)^\alpha (L_t^d)^{1-\alpha} \tag{A.8}$$

$$w_t L_t^d = (1-\alpha) Y_t^d \tag{A.9}$$

$$R_{kt}^d = \frac{\alpha \frac{Y_t^d}{K_{t-1}^d} + (1-\delta) Q_t}{Q_{t-1}} \tag{A.10}$$

$$\phi_t^d N_t^d = Q_t K_t^d \tag{A.11}$$

$$\phi_t^d = \frac{\nu_t^d}{\theta_t^d - \mu_t^d} \tag{A.12}$$

$$\mu_t^d = E_t\{\Lambda_{t,t+1} [(1-\sigma) + \sigma \Theta_{t+1} \psi_{t+1}^{nash} + \sigma(1-\Theta_{t+1}) \theta_{t+1}^d \phi_{t+1}^d](R_{kt+1}^d - R_{t+1}^*)\} \tag{A.13}$$

$$\nu_t^d = E_t\{\Lambda_{t,t+1} [(1-\sigma) + \sigma \Theta_{t+1} \psi_{t+1}^{nash} + \sigma(1-\Theta_{t+1}) \theta_{t+1}^d \phi_{t+1}^d] R_{t+1}^*\} \tag{A.14}$$

$$\psi_t^{nash} = \xi \theta^f \phi_t^f + (1-\xi) \theta_t^d \phi_t^d \tag{A.15}$$

Matching:

$$\Theta_t = \Theta \tag{A.16}$$

Goods market clearing:

$$N_t^d = \sigma(1-\Theta_t)[(R_{kt}^d - R_t^*)\phi_{t-1}^d + R_t^*]N_{t-1}^d + \omega Q_t K_{t-1}^d \quad (A.17)$$

$$N_t^f = \kappa[(R_{kt}^f - R_t^*)\phi_{t-1}^f + R_t^*]N_{t-1}^f + \sigma\Theta_t[(R_{kt}^d - R_t^*)\phi_{t-1}^d + R_t^*]N_{t-1}^d \quad (A.18)$$

$$Y_t = Y_t^d + Y_t^f \quad (A.19)$$

$$L_t = L_t^d + L_t^f \quad (A.20)$$

$$K_t = K_t^d + K_t^f \quad (A.21)$$

$$Y_t - C_t - I_t = (1-\kappa)[R_{kt}^f Q_{t-1} K_{t-1}^f - R_t^* B_{t-1}^{*f}] - \sigma\Theta_t \psi_t^{nash} N_t^d + R_t^* B_{t-1}^* - B_t^* \quad (A.22)$$

$$B_t^{*d} \equiv Q_t K_t^d - N_t^d \quad (A.23)$$

$$B_t^{*f} \equiv Q_t K_t^f - N_t^f \quad (A.24)$$

$$B_t^* \equiv B_t^{*d} + B_t^{*f} \quad (A.25)$$

Capital production:

$$K_t = (1-\delta)K_{t-1} + (1 - Adj_t)I_t \quad (A.26)$$

$$1 = Q_t\left[1 - Adj_t - \Psi^I \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + E_t\left[\Lambda_{t,t+1} Q_{t+1} \Psi^I \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \quad (A.27)$$

Domestic households:

$$w_t = \Psi^L L_t^\varphi, \quad (A.28)$$

$$1 = E_t(\Lambda_{t,t+1} R_{t+1}^d) \quad (A.29)$$

$$\Lambda_{t-1,t} = \beta \frac{\left(C_{t-1} - \Psi^L \frac{L_{t-1}^{1+\varphi}}{1+\varphi}\right)}{\left(C_t - \Psi^L \frac{L_t^{1+\varphi}}{1+\varphi}\right)} \quad (A.30)$$

The above set of 30 equations solve for the following 30 variables:

$$Y_t^d, Y_t^f, Y_t, K_t^f, K_t^d, K_t, L_t^f, L_t^d, L_t, N_t^f, N_t^d, \Theta_t, C_t, I_t$$

$$B_t^*, B_t^{*d}, B_t^{*f}, \phi_t^f, \nu_t^f, \mu_t^f, \phi_t^d, \nu_t^d, \mu_t^d, \psi_t^{nash}$$

$$\Lambda_{t-1,t}, w_t, Q_t, R_{kt}^d, R_{kt}^f, R_t^d$$

We can write down some auxiliary variables of interest:

$$\begin{aligned}\psi_t^d &= \mu_t^d\phi_t^d + \nu_t^d \\ \psi_t^f &= \mu_t^f\phi_t^f + \nu_t^f \\ FDIinflow_t &= \sigma\Theta_t\psi_t^{nash}N_t^d\end{aligned}$$

The set of parameters to be calibrated are given by:

$$\beta, \alpha, \delta, \Psi^I, \theta^f, \kappa, \bar{R}^*, \bar{\theta}^d, \sigma, \bar{\Theta}, \omega, \Psi^L, \varphi, \xi, \chi$$

Appendix B. Derivation of the market Clearing Condition

This section derives the BOP condition in the main text. We start with an individual household's budget constraint:

$$\begin{aligned}&w_tL_t + R_t^dD_{t-1}^d + \int_i s_{it-1}[(1-\sigma)n_{it} + \sigma\Theta_tV_{it}^{nash} + \sigma(1-\Theta_t)V_{it}]di - tr_t + \Pi_t^k \\ =\;& C_t + \int_i s_{it}V_{it}di + D_t^d \end{aligned} \tag{B.1}$$

aggregate them up and we will get

$$w_tL_t + (1-\sigma)\int_i n_{it}di + \sigma\Theta_t\int_i V_{it}^{nash}di - tr_t + \Pi_t^k = C_t \tag{B.2}$$

Integrate over i,

$$w_tL_t + (1-\sigma)[R_{kt}^dQ_{t-1}K_{t-1}^d - R_t^*B_{t-1}^d] + \sigma\Theta_tV_t^{nash} - tr_t + \Pi_t^k = C_t \tag{B.3}$$

which can be written as

$$w_tL_t + R_{kt}^dQ_{t-1}K_{t-1}^d + \sigma\Theta_tV_t^{nash} + \Pi_t^k = C_t + \sigma(R_{kt}^dQ_{t-1}K_{t-1}^d - R_t^*B_{t-1}^d) + R_t^*B_{t-1}^d + tr_t \tag{B.4}$$

Now refer to the equation of N_t^d, we can get

$$w_tL_t + R_{kt}^dQ_{t-1}K_{t-1}^d + \sigma\Theta_tV_t^{nash} + \Pi_t^k = C_t + N_t^d + \sigma\Theta_t(R_{kt}^dQ_{t-1}K_{t-1}^d - R_t^*B_{t-1}^d) + R_t^*B_{t-1}^d \tag{B.5}$$

Now refer to the equation of N_t^f, we can get

$$w_tL_t+R_{kt}^dQ_{t-1}K_{t-1}^d+\sigma\Theta_tV_t^{nash}+\Pi_t^k=C_t+N_t^d+N_t^f-\kappa[R_{kt}^fQ_{t-1}K_{t-1}^f-R_t^*B_{t-1}^f]+R_t^*B_{t-1}^d \quad \text{(B.6)}$$

Since $B_t^f+B_t^d=B_t$, we can write the above equation as

$$w_tL_t+R_{kt}^dQ_{t-1}K_{t-1}^d+\kappa R_{kt}^fQ_{t-1}K_{t-1}^f+\sigma\Theta_tV_t^{nash}+\Pi_t^k=C_t+N_t^d+N_t^f-(1-\kappa)R_t^*B_{t-1}^f+R_t^*B_{t-1} \quad \text{(B.7)}$$

Rewrite it such that on the left hand side we get rid off κ,

$$\begin{aligned}&w_tL_t+R_{kt}^dQ_{t-1}K_{t-1}^d+R_{kt}^fQ_{t-1}K_{t-1}^f+\sigma\Theta_tV_t^{nash}+\Pi_t^k=\\&C_t+N_t^d+N_t^f+(1-\kappa)[R_{kt}^fQ_{t-1}K_{t-1}^f-R_t^*B_{t-1}^f]+R_t^*B_{t-1}\end{aligned} \quad \text{(B.8)}$$

Now deal with the left hand side

$$\begin{aligned}&w_tL_t+(r_{kt}^f+(1-\delta)Q_t)K_{t-1}^f+(r_{kt}^d+(1-\delta)Q_t)K_{t-1}^d+\sigma\Theta_tV_t^{nash}+\Pi_t^k=\\&C_t+N_t^d+N_t^f+(1-\kappa)[R_{kt}^fQ_{t-1}K_{t-1}^f-R_t^*B_{t-1}^f]+R_t^*B_{t-1}\end{aligned} \quad \text{(B.9)}$$

According to the producer's problem, we can get

$$Y_t^d+Y_t^f+\sigma\Theta_tV_t^{nash}+\Pi_t^k=C_t+N_t^d+N_t^f+(1-\kappa)[R_{kt}^fQ_{t-1}K_{t-1}^f-R_t^*B_{t-1}^f]+R_t^*B_{t-1} \quad \text{(B.10)}$$

From the capital producer's profit equation $\Pi_t^K=Q_t[K_t-(1-\delta)K_{t-1}]-I_t$, we can get

$$Y_t+\sigma\Theta_tV_t^{nash}+Q_tK_t=C_t+N_t^d+N_t^f+(1-\kappa)[R_{kt}^fQ_{t-1}K_{t-1}^f-R_t^*B_{t-1}^f]+R_t^*B_{t-1}+I_t \quad \text{(B.11)}$$

From firm's financing equation ($QK=N+B$), we can get

$$Y_t+\sigma\Theta_tV_t^{nash}+N_t^d+N_t^f+B_t=C_t+N_t^d+N_t^f+(1-\kappa)[R_{kt}^fQ_{t-1}K_{t-1}^f-R_t^*B_{t-1}^f]+R_t^*B_{t-1}+I_t \quad \text{(B.12)}$$

In the end, we get

$$Y_t + \underbrace{\sigma\Theta_t V_t^{nash}}_{\text{FDI inflows}} = C_t + I_t + \underbrace{(1-\kappa)[R_{kt}^f Q_{t-1} K_{t-1}^f - R_t^* B_{t-1}^f]}_{\text{FDI outflows}} + R_t^* B_{t-1} - B_t \tag{B.13}$$

which is

$$\underbrace{Y_t - C_t - I_t}_{\text{current account (net exports)}} = \underbrace{\underbrace{(1-\kappa)[R_{kt}^f Q_{t-1} K_{t-1}^f - R_t^* B_{t-1}^f]}_{\text{FDI outflows}} - \underbrace{\sigma\Theta_t V_t^{nash}}_{\text{FDI inflows}}}_{\text{equity financing}} + \underbrace{R_t^* B_{t-1} - B_t}_{\text{debt financing}} \tag{B.14}$$

Appendix C. Calibration strategy

We discuss our calibration strategy of the benchmark model. Given the steady-state spread $R_k^d/R^* = 1.007$, we get $R_k^d = 1.0169$. Furthermore, since $r_k^f = r_k^d \chi^{\frac{1}{\alpha}}$, we get:

$$R_k^f = (R_k^d - (1-\delta))\chi^{\frac{1}{\alpha}} + 1 - \delta = 1.0211.$$

To calibrate the rest of the financial contract, we note that the credit contract conditions need to be satisfied:

$$\phi^f = \frac{\nu^f}{\theta^f - \mu^f} \tag{C.1}$$

$$\mu^f = \frac{1}{R^*}(1 - \kappa + \kappa\theta^f\phi^f)(R_k^f - R^*) \tag{C.2}$$

$$\nu^f = 1 - \kappa + \kappa\theta^f\phi^f \tag{C.3}$$

$$\phi^d = \frac{nu^d}{\bar{\theta}^d - \mu^d} \tag{C.4}$$

$$\mu^d = \beta(1 - \sigma + \sigma\Theta\psi^{nash} + \sigma(1-\Theta)\bar{\theta}^d\phi^d)(R_k^d - R^*) \tag{C.5}$$

$$\nu^d = \beta(1 - \sigma + \sigma\Theta\psi^{nash} + \sigma(1-\Theta)\bar{\theta}^d\phi^d)R^* \tag{C.6}$$

$$\psi^{nash} = \xi\theta^f\phi^f + (1-\xi)\bar{\theta}^d\phi^d \tag{C.7}$$

Rearranging the steady-state version of the evolution of MNC-owned firms' net worth, we get:

$$\frac{N^f}{N^d} = \frac{\sigma\Theta[(R_k^d - R^*)\phi^d + R^*]}{1 - \kappa[(R_k^f - R^*)\phi^f + R^*]} \tag{C.8}$$

The capital ratio is given by:

$$\frac{K^f}{K^d} = \frac{\phi^f}{\phi^d}\frac{N^f}{N^d} \tag{C.9}$$

$$\frac{K^d}{K} = \frac{K_d}{K^d + K^f} = \frac{1}{1 + \frac{K^f}{K^d}} \tag{C.10}$$

Output ratios are given by:

$$\frac{Y^f}{Y^d} = \chi^{\frac{1}{\alpha}}\frac{K^f}{K^d} \tag{C.11}$$

The stock of FDI to output ratio is:

$$\begin{aligned}\frac{sFDI}{Y} = \frac{\psi^f N^f}{Y} &= (\mu^f\phi^f + \nu^f)\frac{N^f}{K^f}\frac{K^f}{Y^f}\frac{Y^f}{Y} \\ &= \frac{(\mu^f\phi^f + \nu^f)}{\phi^f}\frac{\alpha}{R_k^f - (1-\delta)}\left(1 - \frac{1}{1 + \frac{Y^f}{Y^d}}\right)\end{aligned} \tag{C.12}$$

The above 12 equations solve for nine steady-state values $\{\phi^f, \mu^f, \nu^f, \mu^d, \nu^d, \psi^{nash}, \frac{N^f}{N^d}, \frac{K^f}{K^d}, \frac{Y^f}{Y^d}\}$ and three unknown parameters $\{\theta^f, \bar{\theta}^d, \Theta\}$, given known parameters $\{R^*, \kappa, \sigma, \xi, \alpha, \chi, \beta, \delta\}$ and the steady-state values of $\{\phi^d, \frac{K^d}{K}, \frac{sFDI}{Y}\}$.

The evolution of domestic firms' net worth is used to back out the start-up fund parameter ω:

$$\omega = \frac{[1 - \sigma(1-\Theta)[(R_k^d - R^*)\phi^d + R^*]]}{\phi^d}. \tag{C.13}$$

Appendix D. Extensions

Appendix D.1. Model with world interest rate shocks

In the main text, we assume the world interest rate is constant to maintain parsimony of the model. However, the recent literature argues that world interest rate shocks may explain a non-negligible fraction of emerging markets' business cycle (see for example Uríbe and Yue (2006), Neumeyer and Perri (2005)). We show in this appendix that our model can easily allow for world interest rate shocks, but that the shocks will affect FDI and external debt in a way similar to a productivity shock, and so it does not help to account for the changing business cycle pattern of FDI in times of crises.

Specifically, we assume that the world interest rate shock follows an exogenous AR(1) processes as follows:

$$\ln R_t^* \quad = \quad (1-\rho_{R^*})\ln \bar{R}^* + \rho_{R^*}\ln R_{t-1}^* + \epsilon_{R^*t}, \qquad \epsilon_{R^*t} \sim N(0, \sigma_{R^*}^2) \tag{D.1}$$

The innovations of the world interest rate shocks are assumed to be i.i.d, uncorrelated over time and with innovations of productivity and financial shocks.

We need to recalibrate the shock processes. For the interest rate shock, we obtain expected 3-month real interest rate data (including country spreads) for all emerging economies in our sample except Indonesia (because country spread data for Indonesia is not available). The interest rate data is constructed following Neumeyer and Perri (2005). We use secondary market prices of emerging market bonds to recover nominal US dollar interest rates and obtain real rates by subtracting expected US inflation. We obtain data country spread data from Fernández and Gulan (2015) which are retrieved from EMBI Global spread database, and US risk-free interest data is proxied by 3-month T-bill rate. Expected inflation is computed as the average US consumer price index inflation in the current quarter and in the 3 preceding quarters (Both T-bill rate and CPI data available from the St. Louis Federal Reserve FRED database). For each country, we fit an AR(1) process to the interest rate data. The average shock persistence and standard deviation of the innovation are 0.95 and 0.002 respectively. However, since the interest rates are more volatile in crisis periods than normal times, we choose a lower volatility $\sigma_{R^*} = 0.001$.

We calibrate productivity shocks and financial shocks assuming that financial shocks are turned off in non-crisis period and turned on during crises. We select $\rho_A = 0.95$ and use the innovations of the productivity shock process to match the non-crisis standard deviation of output $\sigma_Y = 0.025$, and we obtain $\sigma_A = 0.003$. We keep the financial shock process unchanged. When all shocks are turned on the volatility of output is 3.0%.

Fig. D.5 shows the impulse response to a positive world interest rate shock R_t^* to the small open economy. A rise in the world interest rate increases the costs of borrowing for both domestic and MNC-owned firms, leading to capital outflows and a fall in investment. A drop in Q_t reduces firm net worth. As in the case of a productivity shock, a fall in firm net worth reduces the average size of foreign acquisition, and this effect dominates the price effect (a rise in the acquisition price per unit net worth of domestic firm). As a result, FDI falls. The world interest rate shock, likewise, cannot generate negative comovement between international debt and FDI flows.

Table 3 compares the business cycle statistic generated by the model with world interest rate shocks (last two columns) with the data and also the benchmark model. The correlations between

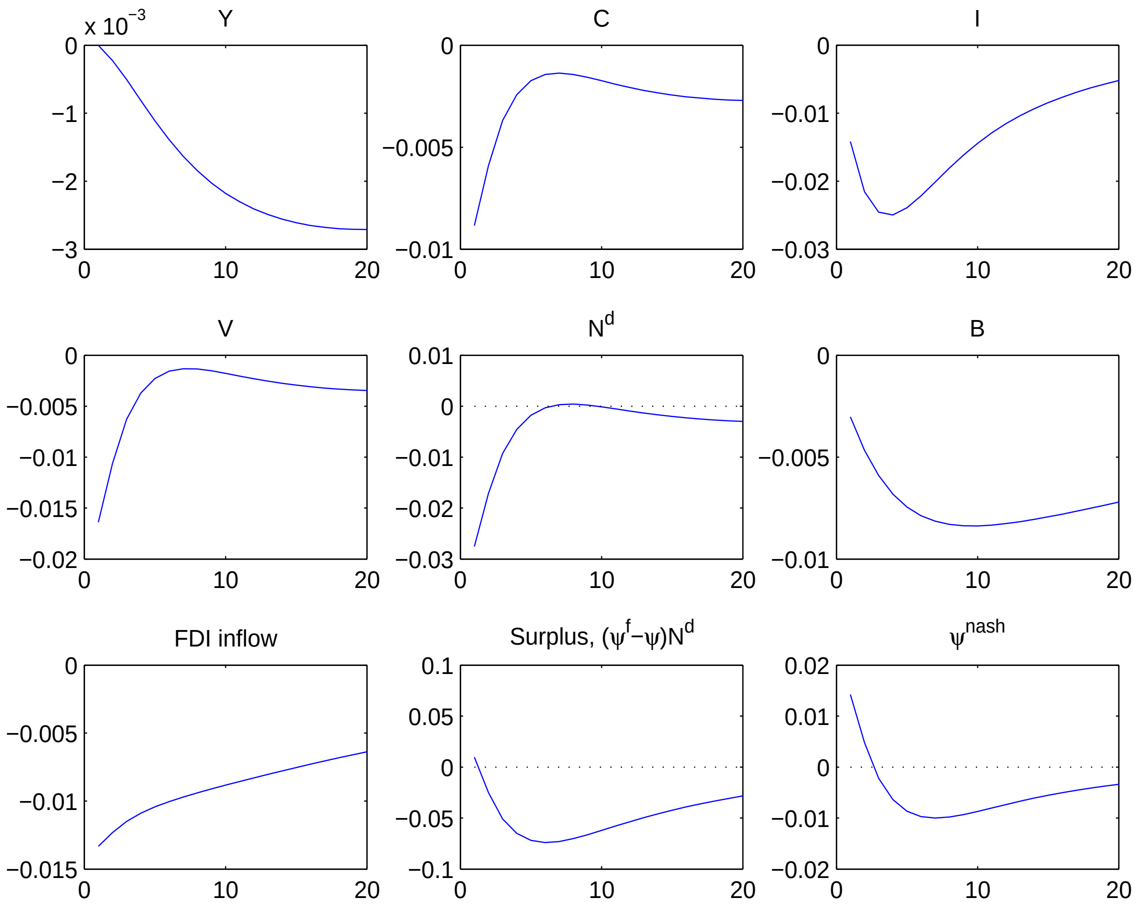

Fig. D.5. Impulse response to a positive world interest rate shock

Table D.4
Business cycle statistics with world interest rates shocks

	Emerging economies		Benchmark model		Model with R^* shocks	
	Normal	Crisis	Normal	Crisis	Normal	Crisis
$\rho(\mathrm{Y}, \mathrm{FDI})$	0.22	-0.10	0.94	-0.21	0.66	0.02
$\rho(\mathrm{Y}, \mathrm{Debt})$	0.22	0.20	0.98	0.66	0.85	0.72
$\rho(\mathrm{I}, \mathrm{FDI})$	0.11	-0.10	0.64	-0.60	0.83	-0.18
$\rho(\mathrm{I}, \mathrm{Debt})$	0.21	0.23	0.78	0.57	0.72	0.58
σ_Y (%)	2.54	3.27	2.53	2.98	2.57	3.03
σ_C/σ_Y	1.06	1.37	1.07	1.55	1.14	1.57
σ_I/σ_Y	3.76	4.69	1.63	3.81	2.97	4.27

Note: Moments of emerging economies are computed by using quarterly data from 1990Q1 to 2012Q3.[19] Source: IFS. The numbers from the model are the averages of 100 series of 2100 periods simulated based on the benchmark calibration. σ_Y denotes the standard deviation of GDP (in percent). σ_i/σ_Y represents the standard deviation relative to that of GDP. $\rho(Y, i)$ is the correlation with GDP. $\rho(I, i)$ is the correlation with investment.

output and FDI in crises time is 0.02, much lower than 0.66 in normal time. The correlations between output and external debt is positive in normal and crises time alike. Moreover, when there is world interest rate shocks, σ_I/σ_Y is much larger than the benchmark model and this brings the volatility of investment much closer to the data.

Appendix D.2. Time-varying matching probability

In the benchmark model, we have kept the matching probability Θ as an exogenous parameter, so the number of foreign acquisition does not change over the business cycle. In reality, however, FDI inflows can change via the intensive margin as well as the extensive margin, i.e. foreign investors may be driven by an increase in the size of the valuation wedge during financial crises and increase their number of acquisitions.

Our model can allow for this. Specifically, we assume that the matching probability increases when the valuation gap widens:[20]

$$\ln\left(\frac{\Theta_t}{\bar{\Theta}}\right) = \Upsilon \ln\left(\frac{(\psi_t^f - \psi_t^d)N_t^d}{(\bar{\psi}^f - \bar{\psi}^d)\bar{N}^d}\right). \tag{D.2}$$

For simplicity, we assume that each domestic firm is small and takes Θ_t as given every period. Then the analytical solution of the firms' value functions is unchanged. The full system is the same as above, except that (A.16) is replaced by (D.2).

We calibrate Υ to match the observed increase in acquisition probability during the Asian financial crisis. Aguiar and Gopinath (2005) finds that this probability increases by 91% between 1996 and 1998. However, Moeller et al. (2005) argue that the crisis coincided with a global wave of cross-border acquisitions, so some of the increase in acquisition probability may not be attributed to the cyclical change. We calibrate the sensitivity parameter $\Upsilon = 0.15$, which corresponds to 16% cumulative increase in acquisition probability in the first 8 quarters after a 1 s.d. financial shock.

With time-varying matching probability, FDI inflows to the small open economy are given by ($FDI_t = \sigma\Theta_t\psi_t^{nash}N_t^d$). Intuitively, the matching probability is increasing in the valuation gap which positively comove with the price effect. So it strengthens the price effect relative to the volume effect. This tends to make $\rho(Y, FDI)$ less positive/ more negative. Table D.5 compares the business cycle statistics of this model with the benchmark model and confirms this intuition.

[20] Smith and Valderrama (2009) endogenize Θ by assuming that foreign investors exert a costly effort in searching for with domestic firms. This yields a matching probability increasing in the valuation wedge.

Table D.5
Business cycle statistics with time-varying matching probability

	Emerging economies		Benchmark model ($\Upsilon = 0$)		Model with variable Θ ($\Upsilon = 0.3$)	
	Normal	Crisis	Normal	Crisis	Normal	Crisis
$\rho(\mathrm{Y}, \mathrm{FDI})$	0.22	-0.10	0.94	-0.21	0.33	-0.26
$\rho(\mathrm{Y}, \mathrm{Debt})$	0.22	0.20	0.98	0.66	0.98	0.66
$\rho(\mathrm{I}, \mathrm{FDI})$	0.11	-0.10	0.64	-0.60	0.06	-0.73
$\rho(\mathrm{I}, \mathrm{Debt})$	0.21	0.23	0.78	0.57	0.78	0.58
σ_Y (%)	2.54	3.27	2.53	2.98	2.53	3.00
σ_C/σ_Y	1.06	1.37	1.07	1.55	1.06	1.33
σ_I/σ_Y	3.76	4.69	1.63	3.81	1.64	3.78

Note: Moments of emerging economies are computed by using quarterly data from 1990Q1 to 2012Q3.[21] Source: IFS. The numbers from the model are the averages of 100 series of 2100 periods simulated based on the benchmark calibration. σ_Y denotes the standard deviation of GDP (in percent). σ_i/σ_Y represents the standard deviation relative to that of GDP. $\rho(Y, i)$ is correlation with GDP. $\rho(I, i)$ is correlation with investment.

Appendix D.3. Model with foreign debt and portfolio choice

This appendix discusses the extended model which allows for endogenous real exchange rate adjustments and a portfolio choice problem of the firms in the small open economy – they can borrow domestically and in international financial markets, facing different interest rates. To do this, we follow the approach of Aoki et al. (2016), which extends the credit contract in Gertler and Karadi (2011).

Firms There is a unit measure of firms $i \in [0, 1]$. Some are owned by domestic households and some by foreign MNCs. Firms acquired by MNCs have superscript f whereas domestic firms have superscript d. These firms have the following production function:

$$y^s_{it} = A^s_t (k^s_{it-1})^{\alpha_K} (l^s_{it})^{\alpha_L} (m^s_{it})^{\alpha_M}, \qquad \text{where } \ s \in \{d, f\} \tag{D.3}$$

where k^s_{it-1} denotes capital, l^s_{it} denotes labor, m^s_{it} denotes imports. We assume $A^d_t = A_t, A^f_t = \chi A_t$, where $\chi \geq 1$ captures higher productivity in MNC-owned firms due to technology spillovers.

A firm has net worth n^s_{it} and can now obtain domestic borrowing b^s_{it} and foreign borrowing $S_t b^{*s}_{it}$ to purchase capital $Q_t k^s_{it}$. S_t denotes the real exchange rate. The firm's balance sheet is $n^s_{it} + b^s_{it} + S_t b^{*s}_{it} = Q_t k^s_{it}$. After production, the firm sells depreciated capital and repays domestic and foreign borrowing with interest. The firm's net worth evolves as follows:

$$n^s_{it} = r^s_{kt} k^s_{it-1} + (1 - \delta) Q_t k^s_{it-1} - R^d_t b^s_{it-1} - R^*_t S_t b^{*s}_{it-1} \tag{D.4}$$

where we r^s_{kt} is the marginal product of capital of a type-s firm, given by:

$$r^s_{kt} k^s_{it-1} \equiv \max_{l^s_{it}, m^s_{it}} \{y^s_{it} - w_t l^s_{it} - S_t m^s_{it}\}.$$

We introduce financial frictions following Aoki et al. (2016). Specifically, assume a firm's ability to divert funds depends on the sources and use of funds. We assume $V^s_{it} \geq \theta^s_t (1 + 0.5\gamma^s (x^s_{it})^2) Q_t k^s_{it}$, where $x^s_{it} \equiv \frac{S_t b^{*s}_{it}}{Q_t k^s_{it}}$ is the fraction of international borrowing. The variable θ^s_t is the fraction of divertible fund when a type-s firm only borrows in the local financial market. A positive γ^s means that a firm can divert a larger fraction of assets when it borrows in international financial markets x^s_{it}. In equilibrium, the incentive constraint must be satisfied so that default will not occur. As in the benchmark model, we assume that the fraction of divertible funds for domestic firms, θ^d_t, follows an exogenous process, but θ^f is a constant. Moreover, we assume that domestic firms face tighter incentive constraints than MNC-owned firms, reflecting the poorer ability of domestically-owned firms to access international financial markets. This means that $\theta^d_t > \theta^f$.

We discuss the values of MNC-owned and domestic firms. At time t, an MNC-owned firm chooses amount to borrow in the financial market. After production takes place, in period $t+1$, there is an exogenous probability, $(1-\kappa)$, a firm exits. The firm will keep accumulating assets until it leaves the industry because it earns a risk-adjusted return that is greater than the world interest rate. The purpose of the firm is to maximize the expected terminal wealth, given by:

$$V^f_{it} \quad = \quad \max E_t \left\{ \Lambda^*_{t,t+1} \frac{S_t}{S_{t+1}} [(1-\kappa) n^f_{it+1} + \kappa V^f_{it+1}] \right\} \tag{D.5}$$

where $\Lambda^*_{t,t+1}$ denotes the stochastic discount factor of foreign investors. For simplicity, assume $\Lambda^*_{t,t+1} = 1/R^*_{t+1}$.[22]

Domestic firms' value function is the same as the one in the benchmark model. A domestic firm exits in a given period with an exogenous probability σ. If it exits the net worth is transferred back to households. If it does not exit, there is a probability Θ it is matched with a foreign MNC. The foreign MNC buys the domestic firm with a price V^{nash}_{it+1} and this value is transferred back to households. The domestic firm treats the probability Θ as exogenous. With probability $\sigma(1-\Theta)$

[22] Suppose foreign investors can save by buying foreign riskfree bonds which pays a return of R^*_t, or by investing in MNC-owned firms in the small open economy. The optimal choice of foreign bonds satisfies $E_t(\Lambda^*_{t,t+1} R^*_{t+1}) = 1$. Our approximation is correct up to first-order approximation. The optimal choice of stocks of MNC-owned domestic firms satisfies

$$E_t \left[\Lambda^*_{t,t+1} \left(R^*_{t+1} - \frac{S_t}{S_{t+1}} \frac{[(1-\kappa) n^f_{it+1} + \kappa V^f_{it+1}]}{V^f_{it}} \right) \right] = 0.$$

the firm continues to operate. The value of a domestic firm is given by:

$$V_{it}^{d} = \max E_t\{\Lambda_{t,t+1}[(1-\sigma)n_{it+1}^{d} + \sigma[\Theta V_{it+1}^{nash} + (1-\Theta)V_{it+1}^{d}]]\}. \tag{D.6}$$

Matching between a domestic firm and an MNC lead to an acquisition. The acquisition value is given by (9).

If the financial constraints are always binding, firms' values are given by:

$$\psi_t^s = \mu_t^s \phi_t^s + \eta_t^s \phi_t^s x_t^s + \nu_t^s, \qquad \text{for } s \in \{d, f\},$$

where $\psi_t^s \equiv V_{it}^s / n_{it}^s$ is the value per unit net worth for type-s firm, and $\phi_t^s \equiv Q_t K_{it}^s / N_{it}^s$ is the leverage for type-s firm. Both ψ_t^s and ϕ_t^s are identical for frms within each type. The variables $\mu_t^f, \eta_t^f, \nu_t^f, \mu_t^d, \eta_t^d, \nu_t^d$ are given by

$$\mu_t^f \equiv E_t\left[\Lambda_{t,t+1}^{*}\Omega_{t+1}^{*}\frac{S_t}{S_{t+1}}(R_{kt+1}^{f} - R_{t+1}^{d})\right], \tag{D.7}$$

$$\eta_t^f \equiv E_t\left[\Lambda_{t,t+1}^{*}\Omega_{t+1}^{*}\frac{S_t}{S_{t+1}}\left(R_{t+1}^{d} - R_{t+1}^{*}\frac{S_{t+1}}{S_t}\right)\right], \tag{D.8}$$

$$\nu_t^f \equiv E_t\left[\Lambda_{t,t+1}^{*}\Omega_{t+1}^{*}\frac{S_t}{S_{t+1}}R_{t+1}^{d}\right], \tag{D.9}$$

$$\mu_t^d \equiv E_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1}^{d} - R_{t+1}^{d})], \tag{D.10}$$

$$\eta_t^d \equiv E_t\left[\Lambda_{t,t+1}\Omega_{t+1}\left(R_{t+1}^{d} - R_{t+1}^{*}\frac{S_{t+1}}{S_t}\right)\right], \tag{D.11}$$

$$\nu_t^d \equiv E_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}^{d}], \tag{D.12}$$

where

$$\Omega_{t+1}^{*} \equiv \Lambda_{t,t+1}^{*}[(1-\kappa) + \kappa\psi_{t+1}^{f}], \tag{D.13}$$

$$\Omega_{t+1} \equiv \Lambda_{t,t+1}[(1-\sigma) + \sigma\Theta\psi_{t+1}^{nash} + \sigma(1-\Theta)\psi_{t+1}], \tag{D.14}$$

$$R_{kt}^{s} \equiv \frac{r_{kt}^{s} + (1-\delta)Q_t}{Q_{t-1}}, \qquad \text{for } s \in \{d, f\}. \tag{D.15}$$

The interpretation of the value function is similar to what is discussed for the benchmark model. There are two differences. First, with endogenous exchange rate movements, MNCs take into account expected exchange rate deviations when they evaluate the returns in the small open economy. Second, since firms can borrow domestically and internationally with imperfect financial markets, the marginal value of a unit of domestic borrowing and international borrowing are different.

The optimal share of foreign borrowing x_t^s is common for every type-s firm, and is given by:

$$x_t^s \;=\; \frac{\mu_t^s}{\eta_t^s}\left(-1+\sqrt{1+\frac{2}{\gamma^s}\left(\frac{\eta_t^s}{\mu_t^s}\right)^2}\right), \qquad \text{for } \; s \in \{d, f\}. \tag{D.16}$$

We briefly discuss the property of x_t^f. First, x_t^f is decreasing in γ^f. When the size of divertible fraction of firm value when it borrows abroad γ^f is larger, each unit of foreign loan tightens the financial constraint by more, so it chooses less foreign loans and x_t^f is smaller. Second, we can show that x_t^f is locally increasing in $\eta_t^{*f} \equiv \eta_t^f/\mu_t^f$.[23] The intuition is as follows. η^f measures the cost advantage (in terms of marginal value of the firm) of borrowing in international financial markets versus in the small open economy. η_t^{*f} weighs this by the total excess marginal value of raising outside funds. When the weighted cost advantage of borrowing in international financial markets rises, the fraction of foreign borrowing increases.

Since the leverage constraint is binding, the optimal leverage for type-s firm is given by:

$$\phi_t^s \;=\; \frac{\nu_t^s}{\theta_t^s\left(1+\frac{\gamma^s}{2}(x_t^s)^2\right)-\mu_t^s-\eta_t^s x_t^s}, \tag{D.17}$$

The linearity of the value functions are still preserved and this allows simple aggregation of the model.

Aggregation Since each type of firms has the same capital to labor ratio, same leverage and same share of external debt, we only need to keep track of the sector level quantities. For $Z \in \{Y, K, L, M, N, B, B^*\}$, we have:

$$Z_t^d = \int_i z_{it}^d di, \qquad Z_t^f = \int_i z_{it}^f di.$$

Furthermore, aggregate quantities are given by $Z_t = Z_t^d + Z_t^f$.

In each period, a fraction $(1-\sigma)$ of domestic firms die. Furthermore, for foreign-owned firms, matches dissolve with an exogenous separation rate $(1-\kappa)$. When a multinational separates from a local firm, it takes the net worth. An equal measure of new domestic firms enter, with start-up funds transferred from domestic households.

[23] See Aoki et al. (2016) for detailed discussion.

Net worth of domestic firms evolves as follow:

$$N_t^d = \sigma(1-\Theta)\left[\left(R_{kt}^d - R_t^d\right)\phi_{t-1} + \left(R_t^d - R_t^*\frac{S_t}{S_{t-1}}\right)\phi_{t-1}x_{t-1} + R_t^d\right]N_{t-1}^d + \omega Q_t K_{t-1}^d \tag{D.18}$$

where $\omega Q_t K_{dt-1}$ is the start-up fund.

Net worth of firms owned by MNCs evolves as follow:

$$\begin{aligned} N_t^f &= \kappa\left[\left(R_{kt}^f - R_t^d\right)\phi_{t-1}^f + \left(R_t^d - R_t^*\frac{S_t}{S_{t-1}}\right)\phi_{t-1}^f x_{t-1}^f + R_t^d\right]N_{t-1}^f \\ &\quad +\sigma\Theta\left[\left(R_{kt}^d - R_t^d\right)\phi_{t-1} + \left(R_t^d - R_t^*\frac{S_t}{S_{t-1}}\right)\phi_{t-1}x_{t-1} + R_t^d\right]N_{t-1}^d \end{aligned} \tag{D.19}$$

Capital goods producers The formulation of capital goods producers is identical to the benchmark model. The evolution of capital is given by:

$$K_t = (1-\delta)K_{t-1} + (1 - Adj_t)I_t, \tag{D.20}$$

where Adj_t are investment adjustment costs, which have a quadratic form as follows:

$$Adj_t = \frac{\Psi^I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2. \tag{D.21}$$

The maximization problem for capital goods producers is:

$$\max E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\Pi_{t+s}^K, \tag{D.22}$$

where $\Pi_t^K = Q_t[K_t - (1-\delta)K_{t-1}] - I_t$. The first order condition for the optimal investment choice is:

$$1 = Q_t\left[1 - Adj_t - \Psi^I\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + E_t\left[\Lambda_{t,t+1}Q_{t+1}\Psi^I\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{D.23}$$

Domestic Households A representative household in the SOE maximizes the following Greenwood et al. (1988) utility:

$$U_t = E_t\sum_{t=0}^{\infty}\beta^t \ln\left(C_t - \Psi^L\frac{L_t^{1+\varphi}}{1+\varphi}\right). \tag{D.24}$$

In each period, the representative household receives wage income, returns from domestic lending and from purchase of domestic equities and the profits of capital producing firms. The household consumes, adjusts their domestic lending and equity portfolios and provides a start-up fund to new domestic firms. These mean that the household faces the following budget constraint:

$$\begin{aligned} & w_t L_t + R_t^d D_{t-1} + \int_i s_{it-1}[(1-\sigma)n_{it} + \sigma\Theta V_{it}^{nash} + \sigma(1-\Theta)V_{it}]di + \Pi_t^k \\ = \ & C_t + \int_i s_{it} V_{it} di + D_t + tr_t. \end{aligned}$$

The first order conditions are:

$$w_t = \Psi^L L_t^{\varphi}, \tag{D.25}$$

$$1 = E_t(\Lambda_{t,t+1} R_{t+1}^d), \tag{D.26}$$

$$\psi_t = \mu_t \phi_t + \eta_t \phi_t x_t + \nu_t. \tag{D.27}$$

where

$$\Lambda_{t-1,t} = \beta \frac{\left(C_{t-1} - \Psi^L \frac{L_{t-1}^{1+\varphi}}{1+\varphi}\right)}{\left(C_t - \Psi^L \frac{L_t^{1+\varphi}}{1+\varphi}\right)}. \tag{D.28}$$

Finally, asset markets clear, which means that $D_t = B_t, s_{it} = 1$, for all i.

Market clearing Goods market clears:

$$C_t + I_t + export_t = Y_t. \tag{D.29}$$

We assume the international demand for domestic good is given by:

$$export_t = \left(\frac{P_t}{e_t P_t^*}\right)^{-\varsigma} \bar{export} = S_t^{\varsigma} \bar{export}. \tag{D.30}$$

The external account is given by:

$$export_t - S_t M_t = (1-\kappa)(R_{kt}^f Q_{t-1} K_{t-1}^f - R_t^d B_{t-1}^f - R_t^* S_t B_{t-1}^{*f}) - \sigma\Theta V_t^{nash} + S_t(R_t^* B_{t-1}^* - B_t^*). \tag{D.31}$$

Finally, there are exogenous shocks to productivity A_t, world interest rates R_t^* and the financial constraint θ_t, and these shock processes are identical to those in the benchmark model. This completes the description of the extended model.

Appendix E. Data

In this appendix, we describe the main variables used in the empirical analysis and the main data sources. We also list the countries in our sample, along with the country crisis years.

Main Variables	**Source**
Real GDP, constant local currency units	World Bank
Capital inflows, foreign direct investment inflows	the updated and extended version of dataset constructed by Alfaro et al. (2014)
Capital inflows, external debt financing inflows	the updated and extended version of dataset constructed by Alfaro et al. (2014)

Country	**Crisis year(s)**
Argentina	1980, 1981, 1982, 1987, 1989, 1995, 2001, 2002, 2007
Brazil	1982, 1983, 1986, 1987, 1990, 1992, 1994, 1999, 2002
Colombia	1982, 1985, 1998
Indonesia	1997, 1998, 2002
Korea	1997, 1998, 2008
Malaysia	1997, 1998
Mexico	1981, 1982, 1994, 1995
Peru	1980, 1981, 1983, 1984, 1985, 1988
Philippines	1981, 1983, 1997, 1998
Thailand	1983, 1997, 1998
Turkey	1982, 1984, 1991, 1996, 2000, 2001