

NON-OBVIOUSNESS AND SCREENING*

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The paper offers a novel justification for the non-obviousness patentability requirement. An innovation involves two stages: *research* results in a technology blueprint, which *development* transforms into a profitable activity. An innovator, who is either efficient or inefficient, must rely on outside finance for the development. Only patented technologies are developed. Strengthening the non-obviousness requirement alleviates adverse selection by discouraging inefficient innovators from doing research, but creates inefficiencies by excluding marginal innovations. We show that it is socially optimal to raise the non-obviousness requirement so as to exclude bad innovators; we also provide several robustness checks and discuss the policy implications.

I. INTRODUCTION

TO BE PATENTABLE, AN INVENTION SHOULD NOT ONLY BE NEW and useful, but also sufficiently different that it would not have been obvious to a ‘Person having ordinary skill in the art’ (Witherspoon [1980]). In 2006, the U.S. Supreme Court triggered a heated debate when, in *KSR vs. Teleflex*, it rejected the ‘rigid’ use of the Teaching-Suggestion-Motivation (TSM) test, replacing it with a ‘realistic’ approach that strengthened the

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non-obviousness requirement and led the Court to invalidate the petitioner's patent (Durie and Lemley [2008]). Following the *KSR* decision, the federal circuit and regional courts have strengthened the bar for non-obviousness (Nock and Gadde [2010]). This can be seen as a response to growing concern that casual inspection of patent applications results in many trivial patents' being granted, leading to costly patent litigation.¹ Lemley [2001] challenged this position, however, justifying such casual inspection as 'rational ignorance.' Observing that the patent value distribution is highly skewed, so that only a small proportion of patents are finally commercialized, he argued that a careful inspection of every patent would be a waste of resources, *ex post* litigation's providing a more cost-effective screening device—pushing this logic further, even casual patent inspection is unnecessary, and the patent system should act as a registry system, as for copyrights.

This calls into question the merit of the non-obviousness requirement. In a recent survey, Denicolò [2008] distinguishes four approaches. The *error cost* approach regards non-obviousness as strengthening the novelty requirement, so as to reduce the probability that the Patent and Trademark Office (PTO) commits type II errors, that is, grants a patent to a technology that is already in the public domain. The *option value* approach² starts from the observation that an innovator has an incentive to implement premature ideas in order to preempt competitors; a non-obviousness requirement then helps counter-balance such a bias. The *sequential innovation* approach³ emphasizes instead the positive externalities exerted by precedent innovators; insisting on non-obviousness then helps protect early innovators against competition from subsequent improvements. The *complementary innovation* approach (Heller and Eisenberg [1998]) builds on the 'tragedy of the anticommons:' coordination failure among patent holders, as well as the risk of opportunistic behavior (hold-up) may prevent the efficient use of key resources when they are subject to multiple rights—a biotech breakthrough may for instance involve dozens of complementary gene patents held by different right holders, which may prevent its development or delay its diffusion (Shapiro [2000]); denying patentability to some of the components can alleviate these problems and increase the incentives to innovate (Ménière [2008]).

Although these are relevant issues, the patent toolbox includes many instruments, such as patent length, patent breadth (lagging or leading), and so forth,⁴ which appear better suited for dealing with the above problems. For example, patent breadth determines the degree to which an innovation

¹ See, e.g., Gleick [2000], Cohen [1994], and Thomas [2001].

² See, e.g., Erkal and Scotchmer [2009].

³ See, e.g., Scotchmer [1996], O'Donoghue [1998], and Hunt [2004].

⁴ For a discussion of the patent toolbox see, e.g., Gilbert and Shapiro [1990], Green and Scotchmer [1995], van Dijk [1996], O'Donoghue [1998], and Denicolò and Zanchettin [2002].

must differ from an already patented one to avoid infringement, and thus circumstances under which subsequent innovators must compensate previous ones; it can thus be tailored to allow for socially desirable improvements whilst protecting the value of the original innovations (Denicolo and Zanchettin [2002]). By contrast, the non-obviousness requirement determines whether the subsequent innovations can be patented or not, and thus constitutes a less direct way of dealing with this issue.⁵

In this paper, we emphasize instead the role of non-obviousness as a screening device, mitigating the agency problems that plague innovators' access to finance. As emphasized by Aghion and Tirole [1994], while the literature often treats an innovator as a 'black-box' representing not only the owner, but also the financier and the developer of an innovation, in practice access to finance is key to the development of innovation. For example, in innovative industries where start-ups and SME's own the technologies⁶ but lack the financial resources needed for their development and commercialization, venture capital activity is significantly and positively associated with patenting rates (Kortum and Lerner [2000]). A major challenge lies in identifying valuable technologies, and this information problem, exacerbated by adverse selection, hinders the access to finance for those innovators who do have valuable patents.⁷ Similar issues arise within firms and groups, when deciding which projects to fund.

Another important feature, emphasized by Kitch [1977], is that patented technologies usually require further improvements in order to become fully operational and, because of their better knowledge of the technology, the original inventors are often essential in this process. Consequently, while the investors claim a stake in the technology, the innovators remain often involved in its development. Thus, investors not only look for valuable technologies, but also seek to cooperate with more competent innovators. The interaction between investors and innovators, however, is also often affected by agency problems, as innovators have private information about their ability.

In this paper, we build on these observations and develop a framework where potential innovators vary in their productivity, which affects both their ability to innovate, and to develop the innovation; an innovator must decide whether to undertake research, in which case he comes up with a technology which may be more or less promising, and requires outside finance for its development. It is socially desirable to encourage only the good innovators, and to finance the development of the most promising

⁵ See Hunt [1999] for a study of the implications of non-obviousness for sequential innovation.

⁶ According to Graham *et al.* [2009], holding patents is a common phenomenon among start-ups and SME's.

⁷ See, e.g., De Meza and Webb [1987], Boadway and Keen [2006], Takalo and Toivanen [2011], and Tykvová [2007].

technologies. The interaction with outside investors is however affected by adverse selection. In this context, non-obviousness acts as a screening device: it helps by preventing inefficient innovators from engaging *ex ante* in wasteful research activities, and contributes in this way to alleviate adverse selection problem at the financing stage. This comes at a cost, however, as *ex post*, the valuable technologies that fail the requirement are no longer developed, due to the threat of imitation. We characterize the optimal non-obviousness requirement and show that, in a simple setting where the innovator is only of two types (efficient or not), it is optimal to fully discourage the inefficient type from engaging in R&D: as long as the inefficient type engages in research with positive probability, the *ex ante* benefit from reducing further this probability dominates the *ex post* cost of restricting the development of marginal technologies.

II. THE MODEL

A risk-neutral innovator, who must decide whether to engage in research activities, can be of two types: good (θ^g , with probability μ) or bad ($\theta^b < \theta^g$, with probability $1 - \mu$); the type θ is the innovator's private information, whereas the probability μ is common knowledge. An innovation involves two stages, research and development. At the research stage, by incurring a private cost R the innovator randomly draws a technology x from the support $[0, +\infty)$, according to a cumulative distribution $F(x, \theta)$ with continuous, differentiable density function $f(x, \theta)$, satisfying the Monotone Likelihood Ratio Property (MLRP): for any $x > y$,

$$(1) \quad \frac{f(x, \theta^g)}{f(x, \theta^b)} > \frac{f(y, \theta^g)}{f(y, \theta^b)}.$$

Once a technology has been drawn, its development requires a monetary cost D and, if successful, yields a profit x . The innovator's ability θ also determines the probability of success; the expected profit from development is thus θx . For welfare analysis purposes, we follow the pioneering work of Loury [1979] and assume that the innovator appropriates the full value of the innovation; social surplus is thus also equal to θx . To simplify the exposition, we normalize the interest rate to zero.⁸

We assume that free-riding concerns are strong enough to prevent unpatented technologies from being developed, and that every technology x is a genuine improvement of the state of art, so that there are no novelty or usefulness issues; the only concern for patentability is non-obviousness which, keeping in line with the literature, is based on the value of innova-

⁸ Introducing a positive interest rate does not affect the analysis and simply amounts to rescaling the cost and benefits of developing an innovation.

tion (Denicolò [2008]): a non-obviousness requirement P means that a technology x is patentable only when $x \geq P$.

Finally, to capture agency problems we assume that the innovator is financially constrained and protected by limited liability.⁹ An investor is thus needed to finance the development stage; there are $N \geq 2$ risk-neutral, competitive investors.

III. ANALYSIS

III(i). *First-Best Benchmark*

We first consider the optimal allocation under complete information (*first-best*). For $i \in \{g, b\}$, let $\tilde{x}^i \equiv D/\theta^i$ denote the threshold above which the technology is worth being developed by an innovator of type θ^i : $\theta^i x - D > 0$ if and only if $x > \tilde{x}^i$. If an innovator of type θ^i does research, the resulting profit and social welfare is

$$(2) \quad W^i = \int_{\tilde{x}^i}^{+\infty} (\theta^i x - D) f(x, \theta^i) dx - R.$$

An innovator of type θ^i should do research if and only if $W^i > 0$. Under complete information, an unregulated market would achieve that:

Proposition 1. Under complete information and in the absence of any non-obviousness requirement, the market outcome yields the first-best allocation.

Proof. As investors are competitive and risk-neutral, at the development stage the innovator fully appropriates the expected net profit $\theta^i x - D$; as a result, the innovator chooses to develop the innovation only if $x \geq \tilde{x}^i$ —and as he must reimburse only $\tilde{x}^i = D/\theta^i$, limited liability is not a problem. Therefore, at the research stage, the innovator's expected benefit from research coincides with W^i , implying that the innovator engages in research when and only when it is desirable to do so. ■

Thus, if the innovator's type were publicly observed, there would be no requirement for a non-obviousness requirement. Competition among investors would ensure that profitable projects (and only those) were developed *ex post*, and only efficient innovators would *ex ante* decide to engage in research activities.

⁹ While for the sake of exposition the research cost R is assumed to be a private cost, the analysis would apply as well to situations where the innovator would have enough resources to support the monetary costs of the research stage, but needs to rely on outside finance for the development stage.

III(ii). *Market Outcome Absent any Non-Obviousness Requirement*

We now consider the more realistic case in which θ is the innovator's private information, and first assume here that any innovation is patentable ($P = 0$).

III(ii)(a). *Development.* We first study the development stage, for a given technology x , when investors expect to face a good type θ^g with probability v . Given the information available, without loss of generality we can restrict attention to contracts offering menus of options, where each option $\zeta = \{T, q, \alpha\}$ stipulates a financing probability q , a transfer T to the innovator, and a profit sharing rule $(\alpha, 1 - \alpha)$ in case of successful development (α representing the innovator's share); because of the innovator's limited liability, the transfers must satisfy $T \geq 0$ (in case development fails) and $T + \alpha x \geq 0$ (in case it succeeds).

We refer to $\zeta^0 = \{0, 0, 0\}$ as the default option (which is for instance relevant if the innovator rejects all offers). Note that any 'null' offer $\{0, 0, \alpha\}$ is equivalent to ζ^0 . We will say that in equilibrium an investor is 'active' if it offers an option, other than a null one, that is accepted with positive probability by at least one type of innovator.

Obviously, a technology $x < \tilde{x}^g (< \tilde{x}^b)$ will never be developed, as this would not be profitable even when the innovator is good. More generally, the following lemma shows that, at the development stage, the market outcome is efficient: when the innovator is of type θ^i , the innovation is developed with probability $q^i = q^{i*}$, where

$$(3) \quad q^{i*} = \begin{cases} 1 & \text{if } x > \tilde{x}^i, \\ 0 & \text{if } x < \tilde{x}^i. \end{cases}$$

However, due to adverse selection, when $x > \tilde{x}^g$ both types of innovator obtain the same share $\tilde{\alpha}(x, v)$ of the *expected* profits θx (whether the innovation is actually developed or not); the share $\tilde{\alpha}(x, v)$ is such that, on average, investors break even:

$$(4) \quad \tilde{\alpha}(x, v) \equiv \frac{\pi^e(x, v)}{\theta^e(v)x},$$

where $\pi^e(x, v) \equiv v(\theta^g x - D) + (1 - v)\max\{\theta^b x - D, 0\}$ denotes the expected profit from the technology, and $\theta^e(v) \equiv v\theta^g + (1 - v)\theta^b$ the expected probability of success.

Lemma 1. At the development stage, when the technology has a value x and the innovator is good ($\theta = \theta^g$) with probability v , the market equilibrium is efficient (i.e., $q^i = q^{i*}$) and such that:

- If $x < \tilde{x}^g$, there is no active investor; the innovator obtains zero profit.
- If instead $x > \tilde{x}^g$:
 - at least one investor offers a contract of the form $(\zeta^i = \{T^i, \alpha^i, q^{i*}\})_{i=g,b}$, where $T^g = 0$, $\alpha^g = \tilde{\alpha}(x, v)$, and $T^b + \alpha^b q^{b*} \theta^b x = \tilde{\alpha}(x, v) \theta^b x$;
 - the expected profit of an innovator of type θ is $\tilde{\alpha}(x, v) \theta x$.

Proof. See Appendix A. ■

Lemma 1 shows that, while the market is efficient at the development stage, a bad innovator obtains the same share $\tilde{\alpha}$ of expected profits as a good innovator, even if his innovation is not developed. If for instance $\tilde{x}^g < x < \tilde{x}^b$, the innovation is developed only when the innovator is good ($q^{g*} = 1, q^{b*} = 0$), and yet a bad innovator gets $T^b = \tilde{\alpha}(x, v) \theta^b x$: investors must ‘buy’ the bad innovator out of the development market.¹⁰ More generally, whilst a good innovator obtains a higher payoff than a bad one, in equilibrium the former subsidizes the latter: as the share is designed so that investors break even on average, we have:

$$\begin{aligned} \tilde{\alpha}(x, v) \theta^g x &< \theta^g x - D, \\ \tilde{\alpha}(x, v) \theta^b x &> \max\{\theta^b x - D, 0\}. \end{aligned}$$

Finally, it is straightforward to check that the share $\tilde{\alpha}(x, v)$ is continuous and increases in x and v :¹¹ a lower share of the profit needs to be left to investors when the value of the technology or the average quality of would-be developers increases.

III(ii)(b). *Research.* We now turn to the research stage, and consider a perfect Bayesian equilibrium where a good innovator does research with probability λ^g whereas a bad innovator does so with probability λ^b . A corollary of the previous Lemma is that, as he obtains a higher payoff at the development stage, a good innovator strictly prefers to undertake research whenever a bad one is willing to do so:

Corollary 1. $\lambda^g = 1$ whenever $\lambda^b > 0$.

Proof. See Appendix B. ■

In what follows, we are interested in equilibria in which a bad innovator undertakes research with probability $\lambda^b = \lambda$ (and thus $\lambda^g = 1$); the investors’ posterior belief is then

¹⁰ A similar buyout scheme implements the optimal allocation in the sequential innovation model of Hopenhayn *et al.* [2006]. Here, however, the investors, rather than subsequent innovators, must buy bad innovators out of the market, in order to finance good ones.

¹¹ See the end of Appendix 6 for a formal proof.

$$v(x, \lambda) \equiv \Pr(\theta = \theta^g | x, \lambda) = \frac{\mu f(x, \theta^g)}{\mu f(x, \theta^g) + \lambda(1 - \mu)f(x, \theta^b)},$$

and the share of profit can be expressed as

$$\alpha^*(x, \lambda) = \tilde{\alpha}(x, v(x, \lambda)).$$

The expected profit of a bad innovator is then equal to

$$\Pi^b(\lambda) \equiv \int_{\tilde{x}^g}^{+\infty} \alpha^*(x, \lambda) \theta^b x f(x, \theta^b) dx - R.$$

As $v(x, \lambda)$ decreases when λ increases, $\alpha^*(x, \lambda)$, and thus $\Pi^b(\lambda)$, increases in λ . Therefore, if $\Pi^b(0) < 0$, a bad innovator would never do research; conversely, if $\Pi^b(1) > 0$, both types of innovator would invest in research. To exclude these trivial situations, we assume:

Assumption 1. $\Pi^b(0) > 0 > \Pi^b(1)$.

It is straightforward to show that Assumption 1 implies that only a good innovator should do research if the innovator's type were publicly observed (that is, $W^g > 0 > W^b$). Furthermore, under this Assumption there exists a unique threshold $\hat{\lambda}$ such that $\Pi^b(\hat{\lambda}) = 0$, or

$$(5) \quad \int_{\tilde{x}^g}^{+\infty} \alpha^*(x, \hat{\lambda}) \theta^b x f(x, \theta^b) dx = R,$$

which characterizes the perfect Bayesian Equilibrium:

Proposition 2. Under Assumption 1:

- from an efficiency standpoint, the innovator should undertake research only when being good;
- however, in the absence of any non-obviousness requirement, there is a unique active PBE outcome, in which the innovator does research with probability 1 when being good and with positive probability $\hat{\lambda}$ when being bad.

Proof. See Appendix C. ■

This Proposition shows that, while the market outcome is efficient *ex post*, at the development stage, it need not be so *ex ante*, at the research stage: due to the limited information available to investors in the development market, good innovators subsidize bad ones; as a result, a bad innovator has excessive incentives to undertake research, and may thus do so

even when it is inefficient. As we will see, introducing a non-obviousness requirement helps alleviate this problem.

III(iii). *Non-Obviousness as a Screening Device*

We now study the impact of a non-obviousness requirement P . Clearly, such a requirement does not affect a technology $x > P$; at the development stage, the continuation equilibrium then remains as described by Lemma 1. Also, as a technology $x < \tilde{x}^g$ is never developed, introducing a patentability requirement $P < \tilde{x}^g$ does not affect the PBE characterized by Proposition 2, and thus has no impact on the overall outcome. Conversely, raising the non-obviousness threshold to $P > \tilde{x}^g$ reduces the return that can be expected from research, as fewer technologies can be developed, and thus tends to discourage a bad innovator from undertaking research. The expected profit of a bad innovator becomes

$$\hat{\Pi}^b(\lambda, P) \equiv \int_P^{+\infty} \alpha^*(x, \lambda) \theta^b x f(x, \theta^b) dx - R,$$

which decreases as P increases; as it tends towards $-R$ when P becomes infinitely larger, the innovator will stop undertaking research for P high enough. Indeed, we have:

Proposition 3. Introducing a non-obviousness requirement P leads the bad innovator to undertake research with probability $\lambda^*(P)$, where:

- $\lambda^*(P) = \hat{\lambda}$ as long as $P \leq \tilde{x}^g$;
- $\lambda^*(P) = 0$ whenever $P \geq x^S$, where the ‘screening’ threshold x^S is such that

$$(6) \quad \hat{\Pi}^b(0, x^S) = 0;$$

- and, for $P \in [\tilde{x}^g, x^S]$, $\lambda^*(P)$ is uniquely defined by $\hat{\Pi}^b(\lambda^*, P) = 0$, and decreases from $\hat{\lambda}$ to 0 as P increases from \tilde{x}^g to x^S .

Proof. See Appendix D. ■

Raising P above \tilde{x}^g involves a trade-off: *ex post*, this prevents the development of marginal technologies (those in the range $[\tilde{x}^g, P]$), which is inefficient and thus reduces welfare; but *ex ante*, this discourages the bad innovator from undertaking research, which enhances welfare. Obviously, it is not optimal to raise P beyond x^S : as the bad innovator no longer undertakes research, raising P further then only worsens welfare, by preventing the development of additional technologies. Conversely, *some* screening is optimal: starting from $P = \tilde{x}^g$, a slight increase in P involves

only a second-order loss of efficiency (as the marginal technologies, for which x is close to \tilde{x}^g , generate only a negligible welfare), but yields a first-order benefit by discouraging the bad innovator (as $\partial\lambda^*/\partial P < 0$ for $P = \tilde{x}^g$). The optimal non-obviousness requirement thus lies in the range $(\tilde{x}^g, x^S]$.

The *MLRP* property (1) actually ensures that, as long as $x < x^S$, the benefit from discouraging the bad innovator from undertaking research dominates the cost of preventing marginal technologies from being developed; hence it is socially optimal to deter fully the bad innovator from undertaking research:

Proposition 4. The socially optimal non-obviousness requirement is $P^* = x^S$.

Proof. See Appendix E. ■

Proposition 4 shows that it is optimal to raise the non-obviousness requirement so as to keep the bad innovator entirely out of the market. It is worth noting that the market cannot achieve this outcome on its own. Suppose for instance that the investors announce that they will not finance any technology $x < x^S$. If it were credible, such a self-regulation would suffice to keep the bad innovator out the market (i.e., $\lambda = 0$). Unfortunately, there is a dynamic inconsistency problem: at the development stage, the investors would then have an incentive to finance the development of any technology $x > x^g$; but anticipating this, a bad innovator would therefore undertake research. Thus, a regulatory intervention is needed to enforce the threshold $P^* = x^S$.

IV. DISCUSSION

IV(i). *Policy Implications*

An immediate policy implication of our analysis is that there is a benefit from maintaining an effective non-obviousness requirement (Meurer and Strandburg [2008]), rather than downgrading the patent system to a copyright system—to be sure, this benefit should be compared with the actual cost of enforcing this requirement.

Several empirical studies highlight problems generated by weak patents.¹² Indeed, a substantial proportion of patents granted in the United States are at risk of being invalidated or narrowed. Determining the precise percentage of dubious patents is difficult, but an investigation of patent overturn rates sheds some light: Allison and Lemley [1998] find for instance that about 46 per cent of the patents challenged on validity grounds between 1989 and 1996 were overturned; and prior to the creation of the

¹² See, e.g., Anton, Greene, and Yao, [2006].

Federal Circuit in 1982, this percentage was closer to 65 per cent. This is in line with our analysis, where a weak non-obviousness requirement leads to excessive entry by bad innovators, and results in a greater proportion of marginal innovations. Having too many marginal innovations is moreover a bad signal, associated with lower social welfare. Raising the bar for non-obviousness can alleviate this problem by discouraging bad innovators from entering the market. Following *KSR vs Teleflex*, the Federal Circuit appears to have taken some steps in that direction.¹³

Our analysis also highlights some determinants of the optimal non-obviousness threshold, $P^* = x^S$; from (6), we have $\frac{\partial P^*}{\partial \mu} = 0$ and:

$$\frac{\partial P^*}{\partial R} = -\frac{1}{\alpha^*(x^S, 0)\theta^b x^S f(x^S, \theta^b)} < 0,$$

$$\frac{\partial P^*}{\partial D} = \frac{\int_{x^S}^{+\infty} \frac{\partial \alpha^*}{\partial D}(x, 0)\theta^b x f(x, \theta^b) dx}{\alpha^*(x^S, 0)\theta^b x^S f(x^S, \theta^b)} < 0,$$

leading to:

Proposition 5. The socially optimal non-obviousness policy P^* decreases as the research cost R or the development cost D increases; it does not depend on the proportion μ of good innovators.

As the objective is to discourage bad innovators, there is less of a need for raising the non-obviousness threshold when research and development costs are important. In the same vein, application fees, which inflate these costs, can also contribute to deter bad innovators. This is in line with Mitchell and Zhang [2012] and Schuett [2013]. Greater financial market frictions, which tend to increase the development cost D ,¹⁴ also lead to weaken the non-obviousness requirement. Conversely, policies aiming at subsidizing research activities should lead to a stricter non-obviousness requirement.

IV(ii). *Robustness Checks*

In this subsection, we present several extensions to discuss the robustness of our insights.

IV(ii)(a). *Development Managers.* We assumed so far that the innovator had to be involved in the development of the technology. Suppose instead

¹³ See, e.g., Nock and Gadde [2010], Mojibi [2010], and Cotropia [2006].

¹⁴ For instance, the development cost can be interpreted as $D = (1 + f)\hat{D}$, where \hat{D} denotes the actual cost and f reflects the market frictions.

that there is a competitive market of risk-neutral development managers, who can develop the technology with a (publicly known) success rate θ^m . Obviously, delegation will never occur if $\theta^m < \theta^b$. If instead $\theta^m > \theta^g$, both types of innovator will delegate the development to a manager; the technology will thus be developed whenever $x > \tilde{x}^m \equiv \frac{D}{\theta^m}$ and, the success rate θ^m being common knowledge, the innovator will obtain the associated profit, $\theta^m x - D$. As the innovator appropriates the welfare he creates, there is no need for government intervention: the innovator will undertake research when it is efficient to do so, as in the complete information case.

We now focus on the more interesting case where $\theta^g > \theta^m > \theta^b$. For the sake of exposition, we moreover suppose here that investors do not observe whether a manager is hired or not (we discuss the case where delegation is observable in Web Appendix A.2), in which case a bad innovator will always delegate the development to a manager. Adapting lemma 1 accordingly, when $P \geq \tilde{x}^g$ the expected profit of a bad innovator becomes

$$\tilde{\Pi}^b(\lambda, P) = \int_P^{+\infty} \tilde{\alpha}^*(x, \lambda) \theta^m x f(x, \theta^b) dx - R,$$

where

$$\tilde{\alpha}^*(x, \lambda) = \frac{v(x, \lambda)(\theta^g x - D) + (1 - v(x, \lambda)) \max\{\theta^m x - D, 0\}}{\Pi^m(x, \lambda)}.$$

It follows that, if $\tilde{\Pi}^b(0, \tilde{x}^g) > 0 > \tilde{\Pi}^b(1, \tilde{x}^g)$, then in the absence of a non-obviousness requirement, the bad innovator would undertake research with positive probability. Our analysis carries over, however: it is optimal to introduce a non-obviousness requirement that is sufficiently stringent to keep the bad innovator out of the market:

Proposition 6. Suppose that $\tilde{\Pi}^b(0, \tilde{x}^g) > 0 > \tilde{\Pi}^b(1, \tilde{x}^g)$. The socially optimal non-obviousness requirement is then $P^* = \tilde{x}^S$, such that $\tilde{\Pi}^b(0, \tilde{x}^S) = 0$.

Proof. See Web Appendix A.1. ■

IV(ii)(b). *Collateral.* Suppose the innovator has some private asset $A < D$, so that, at the development stage, investors can require any collateral $C \leq A$. Increasing the collateral level mitigates the adverse selection problem, and leads to a reduction in the subsidy to the bad innovator. Adapting the proof of Lemma 1, we have:

Lemma 2. At the development stage, the investors ask for maximal collateral (i.e., $C = A$) and the equilibrium is efficient (i.e., when the innovator is of type θ^i , then the technology is developed if $x > \tilde{x}^i$); in addition:

- if $x < \tilde{x}^g$, the innovator obtains zero profit;
- if $\tilde{x}^g < x < \hat{x}(A)$, where $\hat{x}(A) < \tilde{x}^b$ is such that

$$\frac{\theta^b}{\theta^e} v(\theta^g \hat{x} - D) = \left(1 - \frac{\theta^b}{\theta^e}\right) A,$$

the incentive constraints are not binding; a good innovator obtains the full value from the technology, $\theta^g \hat{x} - D$, whereas a bad innovator obtains zero profit,¹⁵

- if $x > \hat{x}(A)$, the incentive constraint of a bad innovator is binding; each type θ^i , where $i = g, b$, obtains an expected profit (net of the collateral A) equal to $\alpha^c \theta^i x - A$, where

$$\alpha^c(x, v) \equiv \tilde{\alpha}(x, v) + \frac{A}{\theta^e(v)x} = \frac{\pi^e(x, v)}{\theta^e(v)x} + \frac{A}{\theta^e(v)x}.$$

This Lemma confirms that the use of a collateral mitigates the adverse selection problem that affects the financing of development, in line with the established literature—see, e.g., Martin [2009]. When the technology is only marginally profitable ($x < \hat{x}$), the bad innovator is no longer subsidized; more generally, the net payoff of a bad innovator decreases (i.e., the subsidy is reduced) as the collateral A increases: for $x > \hat{x}$, using $\alpha^c \theta^e x = \pi^e + A$, this payoff can be expressed as

$$\theta^b \alpha^c x - A = \frac{\theta^b}{\theta^e} [\pi^e + A] - A = \frac{\theta^b}{\theta^e} \pi^e - \left(1 - \frac{\theta^b}{\theta^e}\right) A,$$

which thus decreases as A increases (conversely, the net payoff of a good innovator increases with A).

It remains optimal to keep the bad innovator out of the market. However, as the use of collateral now limits cross-subsidization at the development stage, this can be achieved with a less stringent requirement:

Proposition 7. The optimal threshold P^* , which discourages the bad innovator from undertaking research, decreases as the collateral A increases.

¹⁵ For instance, the following options support an equilibrium, in which the incentive constraints are not binding: $C^g = C^b = A$, $\{q^g = 1, \alpha^g = 1 - (D - A)/\theta^g x, T^g = 0\}$, and $\{q^b = 0, T^b = A\}$. To see that $\hat{x}(A) < \tilde{x}^b$, it suffices to note that, for $x = \tilde{x}^b$ (and $A < D$), a bad innovator obtains a positive payoff by mimicking a good type:

$$\theta^b \alpha^g x - A = \theta^b x - \frac{\theta^b}{\theta^g} D - \left(1 - \frac{\theta^b}{\theta^g}\right) A > \theta^b x - D = 0.$$

Proof. See Web Appendix B. ■

IV(ii)(c). *Pure Signaling.* The analysis also carries over to the case where the ‘non-obviousness’ characteristic x does not affect the value of the innovation, as long as it provides a signal about the innovator’s type. Suppose for instance that the expected profit from developing the technology only depends on the innovator’s type, θ : it is equal to $\theta - D$, with $\theta^g > D > \theta^b$; the variable x only represents the degree of non-obviousness, and still satisfies the *MRLP* property. The equilibrium share of the innovator is now given by

$$\alpha^*(x, \lambda) \equiv \frac{v(x, \lambda)(\theta^g - D)}{\theta^b + v(x, \lambda)(\theta^g - \theta^b)}.$$

Going through the same steps as in our original framework, it can be shown that it is still optimal to set $P = x^S$, where the threshold x^S , designed to keep the bad innovator out of the market, is now defined by

$$\frac{\theta^b}{\theta^g} \int_{x^S}^{+\infty} (\theta^g - D)f(x, \theta^b)dx = R.$$

IV(ii)(d). *Multiple Types.* The analysis can be extended to any number n of types: $\theta \in \Theta = \{\theta^1, \dots, \theta^n\}$, where $\theta^1 < \dots < \theta^n$; let us denote the probability distribution by $\{\mu^1, \dots, \mu^n\}$ and the viability thresholds by $\tilde{x}^i = D/\theta^i$ —that is, it is efficient to develop the technology ($q^{i*} = 1$) if $x > \tilde{x}^i$, and not to develop it ($q^{i*} = 0$) if $x < \tilde{x}^i$.

As before, any type $\theta^j > \theta^i$ undertakes research with probability 1 whenever type θ^i is willing to do so; the ‘active’ types thus constitute a subset of the form $\Theta_k = \{\theta^k, \theta^{k+1}, \dots, \theta^n\}$. If the marginal type θ^k undertakes research with probability λ , then at the development stage the probability distribution becomes $v = \{v^k, \dots, v^n\}$, such that:

$$v^i(\lambda) \equiv \begin{cases} \frac{\lambda\mu^k f(\theta^k, x)}{\lambda\mu^k f(\theta^k, x) + \mu^{k+1} f(\theta^{k+1}, x) + \dots + \mu^n f(\theta^n, x)} & \text{for } i = k, \\ \frac{\mu^i f(\theta^i, x)}{\lambda\mu^k f(\theta^k, x) + \mu^{k+1} f(\theta^{k+1}, x) + \dots + \mu^n f(\theta^n, x)} & \text{for } i > k. \end{cases}$$

The expected type, for a given x , is then $\theta^e(\lambda) = \sum_{i=k}^n v^i(\lambda)\theta^i$. Adapting the proof of Lemma 1 yields:

Lemma 3. The development stage is efficient (i.e., $q^i = q^{i*}$ for every type θ^i that undertakes research) and such that:

- If $x > \tilde{x}^k$, investors offer a pooling contract: $T^* = 0$, $\alpha^*(x, v) = 1 - \frac{D}{\theta^e(v)x}$.
- If $x < \tilde{x}^n$, no active contract is offered.
- For $\tilde{x}^n < x < \tilde{x}^k$, investors offer:
 - for θ^k , a fixed payment $T^{k*} = \alpha^* \theta^k x$;
 - for $\theta^i > \theta^k$, a sharing contract of the form $T^{i*} = 0$, $\alpha^{i*} q^{i*} = \alpha^*$,¹⁶

where α^* is designed so that investors break even: $\alpha^* = \frac{\sum_{i=k}^n \max\{\theta^i x - D, 0\}}{\theta^e(\lambda)x}$.

The proof is similar to that of Lemma 1. The only difference is the ‘buy out’ equilibrium when $\tilde{x}^n < x < \tilde{x}^k$. The investors need to give a share α^* of the expected profit θx to all types of innovator, even when the technology is not developed. For the innovator with the lowest ability, this can be achieved through a fixed payment. For the other innovators whose technology should not be developed (the innovators $i \in \{k + 1, \dots, \hat{i}\}$, say), it is not possible to rely on a fixed payment, as this would not be incentive compatible: all the ‘bad types’ $i < \hat{i}$ would pick the larger transfer $T^i = \alpha^* \theta^i x$ designed for θ^i . The solution consists in approximating a fixed payment with a sharing contract that entails a negligible probability of development, together with a high payoff in case of successful development (see footnote 16).

As in our baseline model, at the development stage ‘bad innovators’ are subsidized by good ones. As a result, bad innovators have excessive incentives to undertake research, and it is optimal to introduce a non-obviousness requirement to keep the worst types of innovator out of the market. It may however be optimal to engage in partial screening. To see this, we now consider a three-type scenario where $\Theta = \{\underline{\theta}, \hat{\theta}, \bar{\theta}\}$, with a probability distribution $\mu = \{\underline{\mu}, \hat{\mu}, \bar{\mu}\}$. Obviously, there is no need for screening when $\underline{W} > 0$ or $\bar{W} < 0$. Furthermore, when $\hat{W} > 0$, the only issue is to discourage the worst type, and the previous analysis shows that it is then optimal to fully keep him out of the market. To focus on the most novel case, we introduce the following assumption:

Assumption 2. $\bar{W} > 0 > \hat{W} > \underline{W}$ and $\underline{\Pi}(0) > 0 > \bar{\Pi}(1)$.

Under Assumption 2, both partial screening and full screening can take place:

¹⁶ When $q^{i*} = 1$, the share is thus α^* ; when instead $q^{i*} = 0$, the contract ‘ $q^i = 0$, $\alpha^{i*} q^{i*} = \alpha^*$ ’ should be interpreted as the limit of ‘ $q^i = \frac{1}{N}$, $\alpha^i = N\alpha^*$ ’ for $N \rightarrow +\infty$. Alternatively, if feasibility reasons constrain financing probabilities to be multiples of some ε , then there exists an ‘ ε -efficient’ equilibrium where, for θ^i and x such that $x < \tilde{x}^i$, $q^i = \varepsilon$ and $\alpha^i = \alpha^*/\varepsilon$.

Proposition 8. Under Assumption 2:

- In the absence of any non-obviousness requirement, the market outcome is such that the worst type of innovator ($\underline{\theta}$) does research with probability λ , whereas the other two ($\hat{\theta}$ and $\bar{\theta}$) do research with probability 1.
- It is optimal to introduce a non-obviousness requirement that keeps the worst type out of the market; depending on the probability distribution of the other types, it may be optimal to keep the middle type out of or in the market.

Proof. See Web Appendix C. ■

The Proposition first confirms that it is optimal to raise the non-obviousness threshold so as to keep the worst type of innovator out of the market. That is, $P \geq \underline{P}$, where the threshold \underline{P} is such that the worst type $\underline{\theta}$ does not do research, whereas the other two types undertake research with probability 1. Consider now raising the threshold beyond \underline{P} . At first, this has no impact on the research decisions (both $\hat{\theta}$ and $\bar{\theta}$ still undertake research with probability 1), and thus reduces welfare, by preventing some technologies from being developed. It is only when it reaches a certain level $\hat{P} > \underline{P}$ that the non-obviousness starts discouraging the middle type $\hat{\theta}$ —and in this range, the previous analysis shows that it is optimal to set the bar high enough (to some level $\bar{P} > \hat{P}$) to keep the middle type out of the market. There are thus two possible candidates for optimal non-obviousness: full screening (i.e., $P = \bar{P}$), which keeps both inefficient types ($\underline{\theta}$ and $\hat{\theta}$) out of the market, or partial screening (i.e., $P = \hat{P}$), which keeps the worst type $\underline{\theta}$ out of the market but lets the middle type $\hat{\theta}$ undertake research. It is straightforward to check that partial screening is optimal when the middle type arises with low probability (that is, when $\hat{\mu}$ is small), as keeping this type out of the market cannot offset, in that case, the cost of preventing the development of technologies $x \in [\underline{P}, \bar{P}]$. Conversely, full screening is optimal when the best type is unlikely (that is, when $\bar{\mu}$ is small).

IV(ii)(e). *Patent Fees.* As noted above, introducing a patent fee F provides an alternative means of screening out the bad innovator as, if a technology cannot be developed in the absence of patent protection, the research cost then becomes $D + F$. In the absence of any non-obviousness requirement, screening out the bad innovator requires a fee F high enough to leave no profit from research to a bad innovator, even if investors anticipate that only a good innovator does research:

$$\int_{\frac{D+F}{\theta^g}}^{+\infty} (\theta^g x - D - F) \frac{\theta^b}{\theta^g} f(x, \theta^b) dx = R.$$

By contrast, relying on non-obviousness requires a threshold $P = x^S$, such that $\hat{\Pi}^b(0, x^S) = 0$, or

$$\int_{x^S}^{+\infty} (\theta^g x - D) \frac{\theta^b}{\theta^g} x f(x, \theta^b) dx = R.$$

Comparing these two conditions yields

$$\int_{\frac{D+F}{\theta^g}}^{+\infty} (\theta^g x - D - F) f(x, \theta^b) dx = \int_{x^S}^{+\infty} (\theta^g x - D) f(x, \theta^b) dx.$$

As the integrand is lower in the LHS than in the RHS, it follows that

$$\frac{D + F}{\theta^g} < x^S.$$

That is, fewer marginal innovations are excluded when relying on a patent fee than on non-obviousness. With non-obviousness, the welfare achieved is

$$W^N = \int_{x^S}^{+\infty} (\theta^g x - D) f(x, \theta^g) dx,$$

whereas with a patent fee it is equal to:

$$\begin{aligned} W^F &= \int_{\frac{D+F}{\theta^g}}^{+\infty} (\theta^g x - D - F) f(x, \theta^g) dx + \int_{\frac{D+F}{\theta^g}}^{+\infty} (1 - \tau) F f(x, \theta^g) dx, \\ &= \int_{\frac{D+F}{\theta^g}}^{+\infty} (\theta^g x - D - \tau F) f(x, \theta^g) dx, \end{aligned}$$

where τ denotes the shadow cost of public funds. As there is less exclusion in the patent fee regime (i.e., $\frac{D+F}{\theta^g} < x^S$), $W^F > W^N$ when τ is small enough: relying on patent fees is then socially desirable. When instead τ is large, $W^F < W^N$ —the comparison between these two instruments should however also account for the cost of enforcing the non-obviousness requirement. Designing an optimal framework that incorporates both of these two instruments constitutes an interesting avenue of research.¹⁷

¹⁷ The applications of an innovation can vary in scale as well as in value. If the scale does not depend on the innovator's type, and patent fees cannot be tailored according to that scale, then non-obviousness may be more effective in targeting the patents that are more likely to be generated by weak innovators.

V. CONCLUDING REMARKS

The rationale of the non-obviousness patentability requirement is controversial and its role is debated. After all, why should society preclude trivial but genuine innovations from being patented? Is it a good idea to add to the burden of PTO's, by imposing an additional check on patent applications? In this paper, we propose a justification for such a non-obviousness requirement. If innovators have private information about their ability to do research, and develop the resulting technologies, the existence of inefficient innovators exerts negative externalities on good ones. In such a context, by excluding trivial patents, a non-obviousness requirement acts as an effective screening instrument. Anticipating that their innovation will be less likely to be patentable, weak innovators will refrain from engaging in R&D, which mitigates adverse selection problems for the development of good innovators' R&D projects.

In recent years, we have seen a trend towards lower patentability requirements. For example, software, which used to fall under copyright protection, has become eligible for patent protection. So are databases and business methods, which are now patentable in some countries, including the U.S., Japan and South Korea. One of the benefits of lowering the patentability requirements is to reduce the examination costs, as PTO's can now examine the applications more casually than before. And while many commentators contend that this merely transfers the burden onto the judicial system, as suggested by the recent surge of patenting and litigation, Lemley [2001] points out that this may still be cost-effective, as only few patents develop a commercial value.

This paper shows however that lowering the patentability requirements may harm social welfare, by exacerbating adverse selection in the access to finance. For instance, many start-ups, lacking the financial resources needed to develop their technologies, rely on the number and quality of their patents for attracting investors. This gives investors useful information about innovators' abilities, an important element for the successful development of their inventions. However, when patentability requirements are weakened, inefficient innovators can enter the market and mimic more efficient ones, making it harder for investors to identify good projects, and harder for good innovators to get financed.

APPENDIX

A. Proof of Lemma 1

We characterize here the equilibria described in Lemma 1. For the sake of exposition, we shall restrict attention to equilibria in which the investors adopt pure strategies (each type of innovator can however randomize over several offers).

Let \tilde{x}^i denote the break-even threshold for the θ^i —innovator, defined by $\theta^i \tilde{x}^i = D$. If $x < \tilde{x}^g$, no active contract can be offered to any type ($q^b = q^g = 0$), as at least one party would get a negative expected profit. From now on, we thus focus on the case $x > \tilde{x}^g$.

Since all parties are risk-neutral, they only care about expected revenues; therefore, there is no scope for stochastic payments or transfers. To facilitate our analysis, we introduce the following notation: for each investor $n \in \{1, 2, \dots, N\}$,

- $J_n = \{j_1, \dots, j_{K_n}\}$ denotes the set of options offered by investor n .
- δ_{n,j_k}^i denotes the probability that a θ^i —innovator accepts the option j_k offered by investor n .
- $\delta_n^i = \sum_{j_k \in J_n} \delta_{n,j_k}^i$ denotes the probability that a θ^i —innovator accepts one of the options offered by investor n .
- Λ_{n,j_k}^i denotes the profit that option j_k yields for investor n when accepted by a θ^i —innovator.

In addition, we introduce the following notation for the equilibrium outcome:

- $\tilde{\Psi}_n$ denotes the expected profit of investor n .
- $\tilde{\zeta}^i \equiv \{\tilde{T}^i, \tilde{q}^i, \tilde{\alpha}^i\}$ denotes the most profitable option for investors, among those adopted by a θ^i —innovator—it can be an option offered by a investor, or the default option $\zeta^0 = \{0, 0, 0\}$.
- $\tilde{\Lambda}^i$ denotes the expected profit that $\tilde{\zeta}^i$ yields for investors, when accepted by a θ^i —innovator, and $\tilde{\Lambda} \equiv v\tilde{\Lambda}^g + (1-v)\tilde{\Lambda}^b$.
- $\tilde{\Upsilon}^i$ denotes the expected profit that $\tilde{\zeta}^i$ yields for a θ^i —innovator.

Lemma 4. In equilibrium, each investor $n \in \{1, 2, \dots, N\}$ obtains $\tilde{\Psi}_n = \tilde{\Lambda}$.

Proof. By construction, for each investor $n \in \{1, 2, \dots, N\}$:

$$\begin{aligned} \tilde{\Psi}_n &= v \sum_{j_k \in J_n} \delta_{n,j_k}^g \Lambda_{n,j_k}^g + (1-v) \sum_{j_k \in J_n} \delta_{n,j_k}^b \Lambda_{n,j_k}^b \\ &\leq v \sum_{j_k \in J_n} \delta_{n,j_k}^g \tilde{\Lambda}^g + (1-v) \sum_{j_k \in J_n} \delta_{n,j_k}^b \tilde{\Lambda}^b \\ &= v\delta_n^g \tilde{\Lambda}^g + (1-v)\delta_n^b \tilde{\Lambda}^b. \end{aligned}$$

Therefore,

$$\sum_{n=1}^N \tilde{\Psi}_n \leq v \sum_{n=1}^N \delta_n^g \tilde{\Lambda}^g + (1-v) \sum_{n=1}^N \delta_n^b \tilde{\Lambda}^b.$$

By construction, $(1 - \sum_{n=1}^N \delta_n^i) \tilde{\Lambda}^i \geq 0$ for $i = g, b$.¹⁸ Therefore, the above inequality implies

$$(7) \quad \sum_{n=1}^N \tilde{\Psi}_n \leq v\tilde{\Lambda}^g + (1-v)\tilde{\Lambda}^b = \tilde{\Lambda}.$$

¹⁸ Either $\sum_{n=1}^N \delta_n^i = 1$, or $\sum_{n=1}^N \delta_n^i < 1$, in which case the default option is selected with positive probability by a type- θ^i innovator, implying $\tilde{\Lambda}^i \geq 0$.

It follows that, for each $n \in \{1, 2, \dots, N\}$:

$$\tilde{\Psi}_n \leq \tilde{\Lambda} - \sum_{\substack{m=1 \\ m \neq n}}^N \tilde{\Psi}_m \leq \tilde{\Lambda},$$

where the last inequality stems from the fact that, by construction, $\tilde{\Psi}_m \geq 0$ for any $m \in \{1, 2, \dots, N\}$.

Assume now that $\tilde{\Psi}_n < \tilde{\Lambda}$, and suppose that investor n deviates and offers $\{\hat{\zeta}^g, \hat{\zeta}^b\}$, where $\hat{\zeta}^i = \{\tilde{T}^i + \varepsilon, \tilde{q}^i, \tilde{\alpha}^i\}$ for ε such that $0 < \varepsilon < \tilde{\Lambda} - \tilde{\Psi}_n$. By construction, $\{\tilde{\zeta}^g, \tilde{\zeta}^b\}$ is incentive compatible, and thus so is $\{\hat{\zeta}^g, \hat{\zeta}^b\}$. Moreover, $\{\hat{\zeta}^g, \hat{\zeta}^b\}$ will be accepted with probability 1, as it gives both types of innovator a strictly higher profit than all other offers. Hence, deviating in this way gives investor n a profit $\tilde{\Lambda} - \varepsilon > \tilde{\Psi}_n$, a contradiction. Therefore, in equilibrium, all investors obtain an expected profit equal to $\tilde{\Lambda}$. ■

Lemma 5. In equilibrium, $\tilde{\Lambda} = 0$.

Proof. As $\tilde{\Psi}_n = \tilde{\Lambda}$ from Lemma 4, condition (7) implies $N\tilde{\Lambda} = \sum_{n=1}^N \tilde{\Psi}_n \leq \tilde{\Lambda}$. As $\tilde{\Lambda} = \tilde{\Psi}_n \geq 0$ by construction, it follows that $\tilde{\Lambda} = 0$. ■

This break-even result for the competitive equilibrium outcome is in line with Rothschild and Stiglitz [1976] and Chassagnon and Chiappori [1997].

Lemma 6. $\tilde{q}^i = q^{i*}$, defined by (3).

Proof. We first show that the options $\tilde{\zeta}^g$ and $\tilde{\zeta}^b$ are efficient (i.e., $\tilde{q}^i = q^{i*}$ for $i = g, b$); by construction, they satisfy:

- the limited liability constraints $\tilde{T}^i \geq 0$ and $\tilde{T}^i + \tilde{\alpha}^i x \geq 0$, for $i \in \{g, b\}$;
- the incentive compatibility constraints:

$$(8) \quad \tilde{T}^g + \tilde{q}^g \tilde{\alpha}^g \theta^g x \geq \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^g x,$$

$$(9) \quad \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^b x \geq \tilde{T}^g + \tilde{q}^g \tilde{\alpha}^g \theta^b x;$$

- and the participation constraints:

$$(10) \quad \tilde{T}^g + \tilde{q}^g \tilde{\alpha}^g \theta^g x \geq 0,$$

$$(11) \quad \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^b x \geq 0.$$

Now, suppose $\tilde{q}^i \neq q^{i*}$ for some $i \in \{g, b\}$, and consider the following deviant offers:

$$\hat{\zeta}^g = \left\{ \hat{T}^g = 0, \hat{q}^g = q^{g*} (= 1), \hat{\alpha}^g = \frac{\tilde{T}^g}{\theta^g x} + \tilde{q}^g \tilde{\alpha}^g + \eta \right\},$$

$$\hat{\zeta}^b = \left\{ \hat{T}^b = \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^b x + \varepsilon, \hat{q}^b = q^{b*}, \hat{\alpha}^b = 0 \right\},$$

where ε and η satisfy $\eta\theta^g x > \varepsilon > \eta\theta^b x > 0$.

The options $\hat{\zeta}^g$ and $\hat{\zeta}^b$ are such that:

- They meet the limited liability conditions: $\hat{T}^g = \hat{\alpha}^b = 0$, $\hat{T}^b > \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^b x \geq 0$ from (11), and $\hat{T}^g + \hat{\alpha}^g x = \frac{\tilde{T}^g}{\theta^g} + \tilde{q}^g \tilde{\alpha}^g x + \eta x > \frac{\tilde{T}^g}{\theta^g} + \tilde{q}^g \tilde{\alpha}^g x$, where the last expression is non-negative:
 - this is obvious if $\tilde{\alpha}^g \geq 0$, as then all terms are non-negative;
 - if instead $\tilde{\alpha}^g < 0$, then $\frac{\tilde{T}^g}{\theta^g} + \tilde{q}^g \tilde{\alpha}^g x \geq \tilde{T}^g + \tilde{\alpha}^g x \geq 0$, where the first inequality stems from $\tilde{q}^g \leq 1$, $\tilde{\alpha}^g < 0$, $\tilde{T}^g \geq 0$ and $\theta^g \leq 1$, and the second one follows from the limited liability properties of $\tilde{\zeta}^g$.
- They moreover strictly satisfy the IC constraints:

$$\begin{aligned} \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^g x &= \left(\frac{\tilde{T}^g}{\theta^g x} + \tilde{q}^g \tilde{\alpha}^g + \eta \right) \theta^g x \\ &> \tilde{T}^g + \tilde{q}^g \tilde{\alpha}^g \theta^g x + \varepsilon \\ &\geq \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^g x + \varepsilon \\ &= \hat{T}^b + \hat{q}^b \hat{\alpha}^b \theta^g x, \end{aligned}$$

where the first inequality stems from $\eta\theta^g x > \varepsilon$ and the second one from (8), and:

$$\begin{aligned} \hat{T}^b + \hat{q}^b \hat{\alpha}^b \theta^b x &= \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^b x + \varepsilon \\ &> \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^b x + \eta\theta^b x \\ &\geq \tilde{T}^g + \tilde{q}^g \tilde{\alpha}^g \theta^b x + \eta\theta^b x \\ &\geq \frac{\theta^b}{\theta^g} \tilde{T}^g + \tilde{q}^g \tilde{\alpha}^g \theta^b x + \eta\theta^b x \\ &= \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^b x, \end{aligned}$$

where the first inequality stems from $\varepsilon > \eta\theta^b x$, the second one from (9), and the third one from $\tilde{T}^g \geq 0$ and $\theta^b < \theta^g$.

- And they attract both types of innovator with probability 1:

$$(12) \quad \hat{Y}^g \equiv \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^g x = \tilde{T}^g + \tilde{q}^g \tilde{\alpha}^g \theta^g x + \eta\theta^g x = \tilde{Y}^g + \eta\theta^g x > \tilde{Y}^g,$$

$$(13) \quad \hat{Y}^b \equiv \hat{T}^b + \hat{q}^b \hat{\alpha}^b \theta^b x = \tilde{T}^b + \tilde{q}^b \tilde{\alpha}^b \theta^b x + \varepsilon = \tilde{Y}^b + \varepsilon > \tilde{Y}^b.$$

To conclude the argument, it suffices to show that these options can bring a positive expected payoff to the deviant investor. This expected payoff can be expressed as

$$\hat{\Lambda} = v\hat{\Lambda}^g + (1-v)\hat{\Lambda}^b,$$

or, using $\hat{\Lambda}^i + \hat{Y}^i = q^{i*}(\theta^i x - D)$, $\tilde{\Lambda}^i + \tilde{Y}^i = \tilde{q}^i(\theta^i x - D)$ and the above expressions:

$$\hat{\Lambda} = \tilde{\Lambda} + v(q^{g*} - \tilde{q}^g)(\theta^g x - D) + (1-v)(q^{b*} - \tilde{q}^b)(\theta^b x - D) - v\eta\theta^g x - (1-v)\varepsilon.$$

As $\tilde{\Lambda} = 0$ from Lemma 5 and $\tilde{q}^i \neq q^{i*}$ for some $i \in \{g, b\}$, this expected payoff is positive for ε, η small enough. ■

Lemma 7. There is cross-subsidization: $\tilde{\Lambda}^g > 0 > \tilde{\Lambda}^b$.

Proof. As $v\tilde{\Lambda}^g + (1-v)\tilde{\Lambda}^b = 0$ from Lemma 5, either $\tilde{\Lambda}^g > 0 > \tilde{\Lambda}^b$, or $\tilde{\Lambda}^g \leq 0 \leq \tilde{\Lambda}^b$. We now rule out the latter case.

Consider first the case $x > \tilde{x}^b$, where $\tilde{q}^g = \tilde{q}^b = 1$ from Lemma 6. Hence, if $\tilde{\Lambda}^g \leq 0 \leq \tilde{\Lambda}^b$, then

$$\begin{aligned} \tilde{Y}^g - \tilde{Y}^b &= (\theta^g x - D - \tilde{\Lambda}^g) - (\theta^b x - D - \tilde{\Lambda}^b) \\ &= (\theta^g - \theta^b)x + (\tilde{\Lambda}^b - \tilde{\Lambda}^g) \\ &\geq (\theta^g - \theta^b)x. \end{aligned}$$

But the incentive compatibility condition (9) implies (using $\tilde{q}^g = \tilde{q}^b = 1$):

$$\tilde{Y}^b = \tilde{T}^b + \tilde{\alpha}^b \theta^b x \geq \tilde{T}^g + \tilde{\alpha}^g \theta^b x = \tilde{Y}^g - \tilde{\alpha}^g (\theta^g - \theta^b)x.$$

Therefore, we have

$$\tilde{\alpha}^g (\theta^g - \theta^b)x \geq \tilde{Y}^g - \tilde{Y}^b \geq (\theta^g - \theta^b)x,$$

and thus $\tilde{\alpha}^g \geq 1$. But then, each type of innovator would obtain more than the whole profit from the innovation, contradicting Lemma 5: we would have: $\tilde{Y}^i \geq \tilde{T}^g + \tilde{\alpha}^g \theta^i x \geq \theta^i x$, as $\tilde{T}^g \geq 0$ and $\tilde{\alpha}^g \geq 1$, and thus $\tilde{\Lambda} \leq -D < 0$.

Consider now the case $x < \tilde{x}^b$, where $\tilde{q}^g = 1$ and $\tilde{q}^b = 0$. Hence, if $\tilde{\Lambda}^g \leq 0 \leq \tilde{\Lambda}^b$, then $\tilde{Y}^b = -\tilde{\Lambda}^g \leq 0$, in which case the participation constraint (11) implies $\tilde{Y}^b = 0$, and thus $\tilde{\Lambda}^g = \tilde{\Lambda}^b = \tilde{Y}^b = 0$, and thus $\tilde{Y}^g = \theta^g x - D > 0$. But then, a bad innovator would obtain a positive payoff from picking $\tilde{\zeta}^g$, contradicting $\tilde{Y}^b = 0$:

- If $\tilde{\alpha}^g > 0$, the limited liability condition $\tilde{T}^g \geq 0$ implies $\tilde{T}^g + \tilde{\alpha}^g \theta^b x > 0$;
- If $\tilde{\alpha}^g \leq 0$, then $\tilde{T}^g + \tilde{\alpha}^g \theta^b x \geq \tilde{T}^g + \tilde{\alpha}^g \theta^g x = \tilde{Y}^g > 0$.

■

Corollary 2. All offers made and accepted in equilibrium are efficient (i.e., such that $q^i = q^{i*}$); in addition, both types of investors obtain a positive payoff and thus choose an option with probability 1, and all offers made and accepted by an innovator of type θ^i are equivalent to $\tilde{\zeta}^i$, for both the investor and that type of innovator.

Proof. We first show that each type of innovator θ^i chooses an option with total probability 1 (and obtains the same payoff $\tilde{Y}^i > 0$ with all the options selected). To see this, note first that $\tilde{Y}^b = q^{b*}(\theta^b x - D) - \tilde{\Lambda}^b \geq -\tilde{\Lambda}^b > 0$; therefore, a bad innovator will indeed choose an option with probability 1, and obtain the same positive payoff \tilde{Y}^b on all options selected. As for a good innovator, note that the incentive compatibility condition yields $\tilde{Y}^g \geq \tilde{T}^b + q^{b*} \tilde{\alpha}^b \theta^g x$. Therefore:

- If $q^{b*} = 0$ or $\tilde{\alpha}^b = 0$, the conclusion follows from $\tilde{T}^b = \tilde{Y}^b > 0$.
- If instead $q^{b*} = 1$ and $\tilde{\alpha}^b \neq 0$, then:
 - If $\tilde{\alpha}^b > 0$, the conclusion follows from $\tilde{T}^b + q^{b*} \tilde{\alpha}^b \theta^g x \geq \tilde{\alpha}^b \theta^g x > 0$;

- If instead $\tilde{\alpha}^b < 0$, the conclusion follows from $\tilde{T}^b + q^{b*} \tilde{\alpha}^b \theta^g x > \tilde{T}^b + \tilde{\alpha}^b x \geq 0$, where the last inequality stems from limited liability.

We thus have $\sum_{n=1}^N \sum_{j_k \in J_n} \delta_{n,j_k}^g = \sum_{n=1}^N \sum_{j_k \in J_n} \delta_{n,j_k}^b = 1$, and:

$$\begin{aligned} 0 &= \sum_{n=1}^N \tilde{\Psi}_n = \sum_{n=1}^N \left\{ v \sum_{j_k \in J_n} \delta_{n,j_k}^g \Lambda_{n,j_k}^g + (1-v) \sum_{j_k \in J_n} \delta_{n,j_k}^b \Lambda_{n,j_k}^b \right\} \\ &= v \left\{ \sum_{n=1}^N \sum_{j_k \in J_n} \delta_{n,j_k}^g q_{n,j_k}^g (\theta^g x - D) - \tilde{Y}^g \right\} \\ &\quad + (1-v) \left\{ \sum_{n=1}^N \sum_{j_k \in J_n} \delta_{n,j_k}^b q_{n,j_k}^b (\theta^g x - D) - \tilde{Y}^b \right\}. \end{aligned}$$

However, we also have

$$0 = \tilde{\Lambda} = v \{ q^{g*} (\theta^g x - D) - \tilde{Y}^g \} + (1-v) \{ q^{b*} (\theta^b x - D) - \tilde{Y}^b \}.$$

Subtracting these two equalities yields

$$0 = v \left(q^{g*} - \sum_{n=1}^N \sum_{j_k \in J_n} \delta_{n,j_k}^g q_{n,j_k}^g \right) (\theta^g x - D) + (1-v) \left(q^{b*} - \sum_{n=1}^N \sum_{j_k \in J_n} \delta_{n,j_k}^b q_{n,j_k}^b \right) (\theta^b x - D),$$

and thus, as $(q^{i*} - q_{n,j_k}^i)(\theta^i x - D) \geq 0$, $q_{n,j_k}^i = q^{i*}$ for every type $i = g, b$, every investor $n = 1, \dots, N$, and any option j_n selected with positive probability by θ^i .

To conclude the argument, it suffices to note that, by construction, each offer accepted by θ^i must give the same payoff \tilde{Y}^i to that type of innovator; but as the offer must moreover be efficient, it also gives the same payoff $\tilde{\Lambda}^i = q^{i*}(\theta^i x - D) - \tilde{Y}^i$ to the investor. ■

Lemma 8. $\tilde{T}^g = 0$ and $\tilde{\alpha}^g > 0$.

Proof. Suppose that $\tilde{T}^g > 0$, and consider the following deviant offers:

$$\begin{aligned} \hat{\zeta}^g &= \left\{ \hat{T}^g = 0, \hat{q}^g = \tilde{q}^g (= q^{g*} = 1), \hat{\alpha}^g = \frac{\tilde{T}^g}{\theta^g x} + \tilde{\alpha}^g + \eta \right\}, \\ \hat{\zeta}^b &= \left\{ \hat{T}^b = \hat{\alpha}^g \theta^b x + \varepsilon, \hat{q}^b = \tilde{q}^b (= q^{b*}), \hat{\alpha}^b = 0 \right\}, \end{aligned}$$

where ε and η satisfy $0 < \varepsilon < \eta(\theta^g - \theta^b)\tilde{q}^g x$. These options are such that:

- They meet the limited liability conditions, as $\hat{T}^g = \hat{\alpha}^b = 0$, and:

$$\begin{aligned} \hat{\alpha}^g x &= \frac{\tilde{T}^g}{\theta^g} + \tilde{\alpha}^g x + \eta x > \tilde{T}^g + \tilde{\alpha}^g x \geq 0, \\ \hat{T}^b &= \hat{\alpha}^g \theta^b x + \varepsilon = \left(\frac{\tilde{T}^g}{\theta^g x} + \tilde{\alpha}^g + \eta \right) \theta^b x + \varepsilon > \theta^b \left(\frac{\tilde{T}^g}{\theta^g} + \tilde{\alpha}^g x \right) \geq \theta^b (\tilde{T}^g + \tilde{\alpha}^g x) \geq 0. \end{aligned}$$

- They strictly satisfy the incentive compatibility constraints:

$$\begin{aligned} \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^g x &= \left(\frac{\tilde{T}^g}{\theta^g x} + \tilde{\alpha}^g + \eta \right) \theta^g x \\ &= \tilde{T}^g + \tilde{\alpha}^g \theta^g x + \eta \theta^g x \\ &> \frac{\theta^b}{\theta^g} (\tilde{T}^g + \tilde{\alpha}^g \theta^g x) + \eta \theta^b x + \varepsilon \\ &= \hat{T}^b + \hat{q}^b \hat{\alpha}^b \theta^b x, \end{aligned}$$

where the inequality stems from $\eta(\theta^g - \theta^b)\tilde{q}^g x > \varepsilon$, $\theta^g > \theta^b$, and $\tilde{T}^g + \tilde{\alpha}^g \theta^g x$ from (10), and:

$$\begin{aligned} \hat{T}^b + \hat{q}^b \hat{\alpha}^b \theta^b x &= \hat{\alpha}^g \theta^b x + \varepsilon \\ &= \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^b x + \varepsilon \\ &> \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^b x. \end{aligned}$$

- Finally, we have

$$\hat{Y}^g - \tilde{Y}^g = (\hat{T}^g + \hat{\alpha}^g \theta^g x) - (\tilde{T}^g + \tilde{\alpha}^g \theta^g x) = \eta \theta^g x > 0,$$

and:

$$\begin{aligned} \tilde{Y}^b &= \tilde{T}^b + q^{b*} \tilde{\alpha}^b \theta^b x \\ &\geq \tilde{T}^g + \tilde{\alpha}^g \theta^b x \\ &= \hat{T}^g + \hat{\alpha}^g \theta^g x + \left(1 - \frac{\theta^b}{\theta^g}\right) \tilde{T}^g - (\eta \theta^b x + \varepsilon) \\ &= \hat{Y}^b + \left(1 - \frac{\theta^b}{\theta^g}\right) \tilde{T}^g - (\eta \theta^b x + \varepsilon), \end{aligned}$$

where the inequality stems from (9). Therefore, the option $\hat{\zeta}^g$ attracts the good innovator with probability 1 and, using Lemma 7 and $\hat{\Lambda}^i + \tilde{Y}^i = \tilde{\Lambda}^i + \tilde{Y}^i = q^{i*}(\theta^i x - D)$, for ε, η small enough we have:

$$\begin{aligned} \hat{\Lambda}^g &= \tilde{\Lambda}^g - \eta \theta^g x > 0, \\ \hat{\Lambda} &= v \hat{\Lambda}^g + (1-v) \hat{\Lambda}^b = \tilde{\Lambda} + \left(1 - \frac{\theta^b}{\theta^g}\right) \tilde{T}^g - \eta \theta^g x - (\eta \theta^b x + \varepsilon) > \tilde{\Lambda} > 0. \end{aligned}$$

As $\hat{Y}^g > \tilde{Y}^g$, the option $\hat{\zeta}^g$ attracts the good innovator with probability 1; therefore, if the deviating investor also attracts the bad innovator with probability p , his expected payoff is

$$\hat{\Psi} = v \hat{\Lambda}^g + (1-v) p \hat{\Lambda}^b,$$

which is positive:

- if $\hat{\Lambda}^b \geq 0$, this follows from $\hat{\Psi} \geq v\hat{\Lambda}^g > 0$;
- if instead $\hat{\Lambda}^b < 0$, this follows from

$$\hat{\Psi} = v\hat{\Lambda}^g + (1-v)p\hat{\Lambda}^b > v\hat{\Lambda}^g + (1-v)\hat{\Lambda}^b = \hat{\Lambda} > 0.$$

The deviation is therefore profitable, contradicting the assumption $\tilde{T}^g > 0$. To conclude the argument, it suffices to note that $\tilde{Y}^g > 0$ (see proof of Corollary 2) then implies $\tilde{\alpha}^g > 0$. ■

Lemma 9. $\tilde{T}^b + q^{b*}\tilde{\alpha}^b\theta^b x = \tilde{\alpha}^g\theta^b x$.

Proof. From Lemmas 6 and 8, the IC constraints are:

$$\begin{aligned} \tilde{\alpha}^g\theta^g x &\geq \tilde{T}^b + q^{b*}\tilde{\alpha}^b\theta^b x, \\ \tilde{T}^b + q^{b*}\tilde{\alpha}^b\theta^b x &\geq \tilde{\alpha}^g\theta^b x. \end{aligned}$$

Suppose now that $\tilde{T}^b + q^{b*}\tilde{\alpha}^b\theta^b x (= \tilde{Y}^b) > \tilde{\alpha}^g\theta^b x$, and consider the following deviant offers:

$$\begin{aligned} \hat{\zeta}^g &= \{\hat{T}^g = 0, \hat{q}^g = \tilde{q}^g (= q^{g*} = 1), \hat{\alpha}^g = \tilde{\alpha}^g + \eta\}, \\ \hat{\zeta}^b &= \{\hat{T}^b = \tilde{\alpha}^g\theta^b x + \varepsilon, \hat{q}^b = \tilde{q}^b (= q^{b*}), \hat{\alpha}^b = 0\}, \end{aligned}$$

where ε and η satisfy $0 < \varepsilon < \eta(\theta^g - \theta^b)x$. These options are such that:

- They meet the limited liability conditions, as $\hat{T}^g = \hat{\alpha}^b = 0$, $\hat{\alpha}^g > \tilde{\alpha}^g > 0$, and $\hat{T}^b > \hat{\alpha}^g\theta^b x > 0$.
- They strictly satisfy the incentive compatibility constraints:

$$\begin{aligned} \hat{T}^g + \hat{q}^g\hat{\alpha}^g\theta^g x &= \tilde{\alpha}^g\theta^g x + \eta\theta^g x \\ &> \tilde{\alpha}^g\theta^b x + \eta\theta^b x + \varepsilon \\ &= \hat{T}^b + \hat{q}^b\hat{\alpha}^b\theta^b x, \end{aligned}$$

where the inequality stems from $\eta(\theta^g - \theta^b)x > \varepsilon$ and $\theta^g > \theta^b$, and:

$$\begin{aligned} \hat{T}^b + \hat{q}^b\hat{\alpha}^b\theta^b x &= \tilde{\alpha}^g\theta^b x + \varepsilon \\ &= \hat{T}^g + \hat{q}^g\hat{\alpha}^g\theta^b x + \varepsilon \\ &> \hat{T}^g + \hat{q}^g\hat{\alpha}^g\theta^b x. \end{aligned}$$

- Finally, we have

$$\hat{Y}^g - \tilde{Y}^g = (\hat{\alpha}^g - \tilde{\alpha}^g)\theta^g x = \eta\theta^g x > 0,$$

and

$$\hat{Y}^b - \tilde{Y}^b = -(\tilde{Y}^b - \tilde{\alpha}^g\theta^b x)\tilde{T}^b + (\eta\theta^b x + \varepsilon),$$

where the first term is positive by assumption. Therefore, using Lemma 7 and $\hat{\Lambda}^i + \hat{\Upsilon}^i = \tilde{\Lambda}^i + \tilde{\Upsilon}^i = q^{i*}(\theta^i x - D)$, for ε, η small enough we have:

$$\begin{aligned} \hat{\Lambda}^g &= \tilde{\Lambda}^g - \eta\theta^g x > 0, \\ \hat{\Lambda} &= v\hat{\Lambda}^g + (1-v)\hat{\Lambda}^b = \tilde{\Lambda} + \left(1 - \frac{\theta^b}{\theta^g}\right)\tilde{T}^g - \eta\theta^g x - (\eta\theta^b x + \varepsilon) > \tilde{\Lambda} > 0. \end{aligned}$$

As $\hat{\Upsilon}^g > \tilde{\Upsilon}^g$, the option $\hat{\zeta}^g$ attracts the good innovator with probability 1; therefore, if the deviating investor also attracts the bad innovator with probability p , his expected payoff is

$$\hat{\Psi} = v\hat{\Lambda}^g + (1-v)p\hat{\Lambda}^b,$$

which is positive:

- if $\hat{\Lambda}^b \geq 0$, this follows from $\hat{\Psi} \geq v\hat{\Lambda}^g > 0$;
- if instead $\hat{\Lambda}^b < 0$, this follows from

$$\hat{\Psi} = v\hat{\Lambda}^g + (1-v)p\hat{\Lambda}^b > v\hat{\Lambda}^g + (1-v)\hat{\Lambda}^b = \hat{\Lambda} > 0.$$

The deviation is therefore profitable, contradicting the assumption $\tilde{T}^b + q^{b*}\tilde{\alpha}^b\theta^b x (= \tilde{\Upsilon}^b) > \tilde{\alpha}^g\theta^b x$. ■

We now complete the characterization of the candidate competitive equilibria. From Lemmas 6, 8 and 9, the equilibrium contracts must satisfy: $\tilde{q}^g = 1$, $\tilde{T}^g = 0$, $\tilde{q}^b = q^{b*}$, and $\tilde{T}^b + q^{b*}\tilde{\alpha}^b\theta^b x = \tilde{\alpha}^g\theta^b x$. The equilibrium share $\tilde{\alpha}^g$ is then determined by the break-even condition¹⁹ of the investors. More precisely:

- Case 1: $\tilde{x}^g < x < \tilde{x}^b$ (buyout). In that case, $\tilde{q}^b = 0$; $\tilde{\alpha}^b$ is thus irrelevant, and $\tilde{T}^b = \tilde{\alpha}^g\theta^b x$: the investors ‘buy’ the bad innovator out of the market. The investors’ break-even condition then yields

$$v[(1 - \tilde{\alpha}^g)\theta^g x - D] - (1-v)\tilde{\alpha}^g\theta^b x = 0,$$

or

$$\tilde{\alpha}^g = \tilde{\alpha}(x, v) = v \frac{\theta^g x - D}{\theta^e(v)x}.$$

- Case 2: $x > \tilde{x}^b$ (pooling). In that case, both types of innovator are financed: $\tilde{q}^b = \tilde{q}^g = 1$. The investors’ break-even condition then yields

$$0 = v[(1 - \tilde{\alpha}^g)\theta^g x - D] + (1-v)[(1 - \tilde{\alpha}^b)\theta^b x - \tilde{T}^b - D],$$

which, using $\tilde{T}^b + \tilde{\alpha}^b\theta^b x = \tilde{\alpha}^g\theta^b x$, can be rewritten as

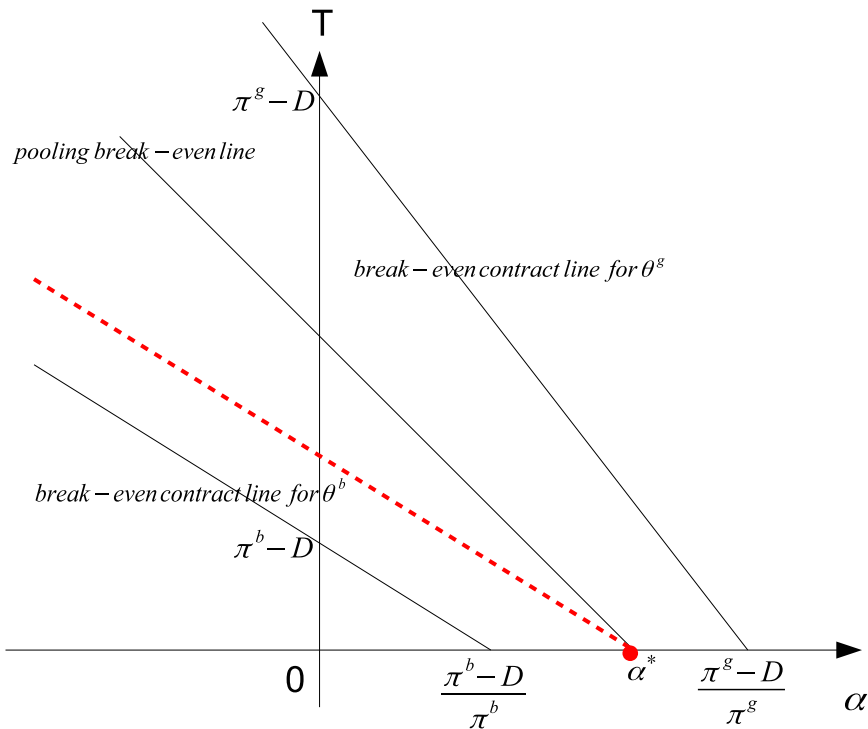
¹⁹ See Lemma 5.

$$0 = (1 - \tilde{\alpha}^g)\theta^e(v)x - D,$$

or

$$\tilde{\alpha}^g = \tilde{\alpha}(x, v) = 1 - \frac{D}{\theta^e(v)x}.$$

Any $\tilde{\zeta}^b = \{\tilde{T}^b, 1, \tilde{\alpha}^b\}$ satisfying $\tilde{T}^b + \tilde{\alpha}^b\theta^b x = \tilde{\alpha}(x, v)\theta^b x$ and the limited liability conditions (that is, $\tilde{T}^b, \tilde{T}^b + \tilde{\alpha}^b x \geq 0$) is a possible candidate equilibrium outcome. Graphically, in the (α, T) plane the equilibrium option for the good innovator is located at the point $(\alpha^*, 0)$, whereas the admissible options for the bad innovator lie anywhere on the dashed line, which starts from the same point $(\alpha^*, 0)$ and parallels the break-even line for a good innovator.



Note that, in both cases, the good type subsidizes the bad one; this is obvious when $x < \tilde{x}^b$, since a bad innovator then obtains $\tilde{T}^b > 0$ even though his innovation does not get developed, and still holds when $x > \tilde{x}^b$, as $\theta^b < \theta^e(v)$ implies

$$(1 - \tilde{\alpha})\theta^b x < (1 - \tilde{\alpha})\theta^e(v)x = D,$$

or

$$(14) \quad \tilde{\alpha}\theta^b x > \theta^b x - D.$$

Finally, we conclude the proof of Lemma 1 by showing that there indeed exists an equilibrium in which all investors offer $\{\tilde{\zeta}^g, \tilde{\zeta}^b\}$, supported by the following strategies:

- if at least one active investor offers $\{\tilde{\zeta}^g, \tilde{\zeta}^b\}$, and no investor offers more than \tilde{Y}^i to a type θ^i , that type of innovator picks the investor with the lowest n among those that offer $\{\tilde{\zeta}^g, \tilde{\zeta}^b\}$;
- if an investor offers more than \tilde{Y}^i to a type θ^i , that type of innovator picks randomly an investor among those that offer the best value for that type.

These continuation strategies for the innovator prevents in particular deviations that simply consist in dropping the loss-making option $\tilde{\zeta}^b$: in equilibrium, all investors offer $\{\tilde{\zeta}^g, \tilde{\zeta}^b\}$ and the innovator thus picks the first one; but if the first one were to drop $\tilde{\zeta}^b$ (and offer only $\tilde{\zeta}^g$), then the innovator would turn to the second investor.

To attract an innovator of type θ^i , a deviating investor must therefore offer more than \tilde{Y}^i to that type. It is straightforward to check that it cannot be profitable to attract only θ^b : since the equilibrium contract $\tilde{\zeta}^b$ is efficient ($\tilde{q}^b = q^{b*}$), offering more than \tilde{Y}^b would then result in a loss since $\tilde{\lambda}^b$ is already negative. Furthermore, it is impossible to attract θ^g without attracting θ^b :

$$\begin{aligned} \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^b x &= \frac{\theta^b}{\theta^g} \left[\frac{\theta^g}{\theta^b} \hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^g x \right] \\ &\geq \frac{\theta^b}{\theta^g} (\hat{T}^g + \hat{q}^g \hat{\alpha}^g \theta^g x) \\ &> \frac{\theta^b}{\theta^g} \alpha^* \theta^g x \\ &= \alpha^* \theta^b x = \tilde{T}^b + q^{b*} \tilde{\alpha}^b \theta^b x. \end{aligned}$$

But then, since the equilibrium options $\{\tilde{\zeta}^g, \tilde{\zeta}^b\}$ are efficient ($\tilde{q}^i = q^{i*}$ for $i \in \{g, b\}$), offering more than \tilde{Y}^i to at least one type θ^i will result in a loss, since in equilibrium the investors barely break even.

We conclude with the properties of $\tilde{\alpha}$. The continuity stems directly from the definition given by (4). As for the comparative statics:

- If $x \leq \tilde{x}^b$, then $\theta^b x - D \leq 0$ and

$$\tilde{\alpha}(x, v) = \frac{v(\theta^g x - D)}{\theta^e(v)x} = \frac{1 - \frac{D}{\theta^g x}}{1 + \frac{1-v}{v} \frac{\theta^b}{\theta^g}},$$

where in the last expression, the numerator increases with x and the denominator decreases as v increases.

- If $x > \tilde{x}^b$, then $\alpha^*(x, v) = 1 - \frac{D}{\theta^e(v)x}$, where $\theta^e(v)x$ increases with x and v .

B. Proof of Corollary 1

Suppose that a bad innovator undertakes research with positive probability; then, for a given technology x , he gets financed with probability $q^{b*}(x)$ and, whenever $x > \tilde{x}^g$, receives an expected payment equal to $\tilde{\alpha}(x, v)\theta^i x$; the expected profit from undertaking research is thus

$$\Pi^b = \int_{\tilde{x}^g}^{+\infty} \tilde{\alpha}(x, v)\theta^b x f(x, \theta^b) dx - R = \int_{\tilde{x}^g}^{+\infty} \theta^b t(x) f(x, \theta^b) dx - R,$$

where $t(x) \equiv \tilde{\alpha}(x, v)x$ is positive and increases with x : $t'(x) = \tilde{\alpha}(x, v) + \frac{\partial \tilde{\alpha}}{\partial x}(x, v) > 0$. A good innovator would obtain instead an expected profit equal to

$$\Pi^g = \int_{\tilde{x}^g}^{+\infty} \tilde{\alpha}(x, v)\theta^g x f(x, \theta^g) dx - R = \int_{\tilde{x}^g}^{+\infty} \theta^g t(x) f(x, \theta^g) dx - R.$$

The difference between these two expected profits can be expressed as:

$$\begin{aligned} \Pi^g - \Pi^b &= \int_{\tilde{x}^g}^{+\infty} \{ \theta^g t(x) f(x, \theta^g) - \theta^b t(x) f(x, \theta^b) \} dx \\ &> \int_{\tilde{x}^g}^{+\infty} \{ \theta^b t(x) f(x, \theta^g) - \theta^b t(x) f(x, \theta^b) \} dx \\ &= \int_{\tilde{x}^g}^{+\infty} \theta^b t(x) \{ f(x, \theta^g) - f(x, \theta^b) \} dx \\ &\geq 0, \end{aligned}$$

where the strict inequality stems from $\theta^g > \theta^b$ and $t(x) > 0$, while the last inequality follows from $t'(x) > 0$ and first-order stochastic dominance.

C. Proof of Proposition 2

Assumption 1 implies:

- $\Pi^b(0) > 0$, or (using $\alpha^*(x, 0) = \frac{\theta^g x - D}{\theta^g x}$)

$$\begin{aligned} R &< \int_{\tilde{x}^g}^{+\infty} \alpha^*(x, 0)\theta^b x f(x, \theta^b) dx \\ &= \frac{\theta^b}{\theta^g} \int_{\tilde{x}^g}^{+\infty} (\theta^g x - D) f(x, \theta^b) dx \\ &< \int_{\tilde{x}^g}^{+\infty} (\theta^g x - D) f(x, \theta^g) dx, \end{aligned}$$

where the last inequality stems from $\theta^b < \theta^g$, $\theta^g x - D$ increasing in x and *MLRP*. It follows that a good innovator should undertake research: $W^g < 0$.

- $0 > \Pi^b(1)$, or

$$R > \int_{\tilde{x}^g}^{+\infty} \alpha^*(x, 1)\theta^b xf(x, \theta^b) dx$$

$$\geq \int_{\tilde{x}^g}^{+\infty} \max\{\theta^b x - D, 0\} f(x, \theta^b) dx,$$

where the weak inequality stems from the bad innovator’s being subsidized by the good one at the development stage (see (14)). It follows that a bad innovator should not undertake research: $W^b < 0$.

We now consider the market equilibrium. Let λ^g (resp., λ^b) denote the probability that the innovator undertakes research when being good (resp., being bad).

Suppose first that $\lambda^g < 1$. Corollary 1 implies $\lambda^b = 0$; but then, under Assumption 1 a bad innovator would have an incentive to deviate and undertake research, a contradiction. Therefore, $\lambda^g = 1$.

In the same vein, if $\lambda^b < \hat{\lambda}$ then a bad innovator would have an incentive to undertake research with probability 1, a contradiction; conversely, if $\lambda^b > \hat{\lambda}$ then a bad innovator would have an incentive to undertake research with probability 0, a contradiction. Therefore, the only candidate equilibrium is such that $\lambda^b = \hat{\lambda}$; conversely, $(\lambda^g = 1, \lambda^b = \hat{\lambda})$ constitutes indeed an equilibrium, as the bad innovator is then indifferent between doing research or not—and thus, from Corollary 1, the good innovator is willing to undertake research.

D. Proof of Proposition 3

As already noted, setting $P \leq \tilde{x}^g$ has no impact on the development stage: as in the baseline scenario (i.e., as for $P = 0$), only those technologies such that $x > \tilde{x}^g$ are developed with positive probability and yield a positive profit to the innovator. For $P > \tilde{x}^g$, the expected profit of a bad innovator becomes

$$\hat{\Pi}^b(\lambda, P) \equiv \int_P^{+\infty} \alpha^*(x, \lambda)\theta^b xf(x, \theta^b) dx - R,$$

where $\frac{\partial \hat{\Pi}^b}{\partial P}(\lambda, P) = -\alpha^*(P, \lambda)\theta^b Pf(P, \theta^b) < 0$ and, as $\frac{\partial \alpha^*}{\partial \lambda}(x, \lambda) < 0$:

$$\frac{\partial \hat{\Pi}^b}{\partial \lambda}(\lambda, P) = \int_P^{+\infty} \frac{\partial \alpha^*}{\partial \lambda}(x, \lambda)\theta^b xf(x, \theta^b) dx < 0.$$

Under Assumption 1, $\hat{\Pi}^b(0, \tilde{x}^g) = \Pi^b(0) > 0$. Since $\frac{\partial \hat{\Pi}^b}{\partial P} < 0$ and $\hat{\Pi}^b(0, +\infty) = -R$, there exists a unique threshold $x^S > \tilde{x}^g$ satisfying $\hat{\Pi}^b(0, x^S) = 0$, or (6). Furthermore, in the range $\tilde{x}^g < P < x^S$, we have:

- $\hat{\Pi}^b(0, P) > \hat{\Pi}^b(0, x^S) = 0$;
- $\hat{\Pi}^b(\hat{\lambda}, P) < \hat{\Pi}^b(\hat{\lambda}, \tilde{x}^g) = \Pi^b(\hat{\lambda}) = 0$.

As $\hat{\Pi}^b(\lambda, P)$ decreases as λ increases, it follows that there exists a unique $\lambda^*(P)$ such that $\hat{\Pi}^b(\lambda^*, P) = 0$. Furthermore, by construction we have $\lambda^*(\tilde{x}^g) = \hat{\lambda}$, $\lambda^*(x^S) = 0$, and, in the range $P \in [\tilde{x}^g, x^S]$, the implicit function theorem yields

$$(15) \quad \frac{d\lambda^*}{dP} = - \left. \frac{\frac{\partial \hat{\Pi}^b}{\partial P}}{\frac{\partial \hat{\Pi}^b}{\partial \lambda}} \right|_{\lambda=\lambda^*} = \frac{\alpha^*(P, \lambda^*(P))Pf(P, \theta^b)}{\int_P^{+\infty} \frac{\partial \alpha^*}{\partial \lambda}(x, \lambda^*(P))xf(x, \theta^b)dx} < 0.$$

The end of the proof follows the same step as for Proposition 2: for $\lambda > \lambda^*$, a bad innovator would rather not undertake research, whereas for $\lambda < \lambda^*$, a bad innovator would derive a positive expected profit from undertaking research. Conversely, when $\lambda = \lambda^*$, $\hat{\Pi}^b = 0$ implies that a bad innovator is indifferent between undertaking research or not, and a good innovator is thus willing to undertake research.

E. Proof of Proposition 4

By construction, in the equilibria characterized by Proposition 3, a bad innovator and the investors obtain zero profits; therefore, social welfare coincides with the expected net profit of a good innovator

$$\hat{W}(P) = \mu \left[\int_P^{+\infty} \alpha^*(x, \lambda^*(P))\theta^g xf(x, \theta^g)dx - R \right].$$

For $P < x^S$, differentiating this expression with respect to P leads to

$$\frac{1}{\mu} \frac{d\hat{W}}{dP}(P) = -\alpha^*(P, \lambda^*(P))\theta^g Pf(P, \theta^g) + \int_P^{+\infty} \frac{\partial \alpha^*}{\partial \lambda}(x, \lambda^*(P)) \frac{d\lambda^*}{dP}(P)\theta^g xf(x, \theta^g)dx.$$

Using (15), this can be expressed as:

$$\begin{aligned} \frac{1}{\mu} \frac{d\hat{W}}{dP}(P) &= -\theta^g \frac{f(P, \theta^g)}{f(P, \theta^b)} \alpha^*(P, \lambda^*(P))Pf(P, \theta^b) \\ &\quad + \frac{d\lambda^*}{dP}(P)\theta^g \int_P^{+\infty} \frac{\partial \alpha^*}{\partial \lambda}(x, \lambda^*(P))xf(x, \theta^g)dx \\ &= -\theta^g \frac{f(P, \theta^g)}{f(P, \theta^b)} \frac{d\lambda^*}{dP}(P) \int_P^{+\infty} \frac{\partial \alpha^*}{\partial \lambda}(x, \lambda^*(P))xf(x, \theta^b)dx \\ &\quad + \frac{d\lambda^*}{dP}(P)\theta^g \int_P^{+\infty} \frac{\partial \alpha^*}{\partial \lambda}(x, \lambda^*(P))xf(x, \theta^g)dx \\ &= \frac{d\lambda^*}{dP}(P)\theta^g f(P, \theta^g) \int_P^{+\infty} \frac{\partial \alpha^*}{\partial \lambda}(x, \lambda^*(P)) \left[\frac{f(x, \theta^g)}{f(P, \theta^g)} - \frac{f(x, \theta^b)}{f(P, \theta^b)} \right] xdx. \end{aligned}$$

From the *MLRP* property, $\frac{f(x, \theta^g)}{f(P, \theta^g)} > \frac{f(x, \theta^b)}{f(P, \theta^b)}$ for any $x > P$; as $\frac{\partial \alpha^*}{\partial \lambda} < 0$, and $\frac{d\lambda^*}{dP} < 0$ as long as $\tilde{x}^g < P < x^S$, it follows that $\hat{W}(P)$ strictly increases with P in that range.

By contrast, for $P > x^S$, $\lambda^*(P) = 0$ and thus

$$\frac{d\hat{W}}{dP}(P) = -\mu\alpha^*(P, 0)\theta^g Pf(x, \theta^g) < 0.$$

The socially optimal threshold is thus $P = x^S$.

REFERENCES

- Aghion, P. and Tirole, J., 1994, 'The Management of Innovation.' *The Quarterly Journal of Economics*, 109(4), pp. 1185.
- Allison, J. and Lemley, M., 1998, 'Empirical Evidence on the Validity of Litigated Patents,' *American Intellectual Property Law Association Quarterly Journal*, 26, pp. 185–277.
- Anton, J.; Greene, H. and Yao, D., 2006, 'Policy Implications of Weak Patent Rights.' *Innovation Policy and the Economy*, 6, pp. 1–26.
- Boadway, R. and Keen, M., 2006, 'Financing and Taxing New Firms under Asymmetric Information.' *FinanzArchiv: Public Finance Analysis*, 62(4), pp. 471–502.
- Chassignon, A. and Chiappori, P. A., 1997, 'Insurance under Moral Hazard and Adverse Selection: The Case of Pure Competition,' *DELTA-CREST Document*.
- Cohen, J., 1994, 'Reverse Engineering and the Rise of Electronic Vigilantism: Intellectual Property Implications of Lock-Out Programs,' *Southern California Law Review*, 68, pp. 1091.
- Cotropia, C., 2006, 'Nonobviousness and the Federal Circuit: An Empirical Analysis of Recent Case Law,' *Notre Dame Law Review*, *Forthcoming*.
- De Meza, D. and Webb, D., 1987, 'Too Much Investment: A Problem of Asymmetric Information,' *The Quarterly Journal of Economics*, 102(2), pp. 281.
- Denicolò, V., 2008, 'Economic Theories of the Nonobviousness Requirement for Patentability: A Survey,' *Lewis & Clark Law Review*, 12, pp. 443.
- Denicolo, V. and Zanchettin, P., 2002, 'How Should Forward Patent Protection be Provided?,' *International Journal of Industrial Organization*, 20(6), pp. 801–827.
- Durie, D. and Lemley, M., 2008, 'A Realistic Approach to the Obviousness of Inventions,' *William & Mary Law Review*, 50, pp. 989.
- Erkal, N. and Scotchmer, S., 2009, 'Scarcity of Ideas and R&D Options: Use It, Lose It or Bank It,' *NBER working Paper Series*, available at <http://www.nber.org/papers/w14940>.
- Gilbert, R. and Shapiro, C., 1990, 'Optimal Patent Length and Breadth,' *The RAND Journal of Economics*, pp. 106–112.
- Gleck, J., 2000, 'Patently Absurd,' *New York Times Magazine*, 12, pp. 44–49.
- Graham, S.; Merges, R. P.; Samuelson, P. and Sichelman, T., 2009, 'High Technology Entrepreneurs and the Patent System: Results of the 2008 Berkeley Patent Survey,' *Berkeley Technology Law Journal*, 24(4), pp. 1255.
- Green, J. and Scotchmer, S., 1995, 'On the Division of Profit in Sequential Innovation,' *The RAND Journal of Economics*, pp. 20–33.
- Heller, M. and Eisenberg, R., 1998, 'Can Patents Deter Innovation? The Anticommons in Biomedical Research,' *Science*, 280(5364), pp. 698.
- Hopenhayn, H.; Llobet, G. and Mitchell, M., 2006, 'Rewarding Sequential Innovators: Prizes, Patents, and Buyouts,' *Journal of Political Economy*, 114(6), pp. 1041–1068.
- Hunt, R., 1999, 'Nonobviousness and the Incentive to Innovate: An Economic Analysis of Intellectual Property Reform,' *Federal Reserve Bank of Philadelphia working paper*, pp. 99–3.
- Hunt, R., 2004, 'Patentability, Industry Structure, and Innovation,' *The Journal of Industrial Economics*, 52(3), pp. 401–425.

- Kitch, E., 1977, 'The Nature and Function of the Patent System,' *Journal of Law and Economics*, 20(2), pp. 265–290.
- Kortum, S. and Lerner, J., 2000, 'Assessing the Contribution of Venture Capital to Innovation,' *RAND Journal of Economics*, pp. 674–692.
- Lemley, M., 2001, 'Rational Ignorance at the Patent Office,' *Northwest University Law Review*, 95, pp. 1495.
- Loury, G., 1979, 'Market Structure and Innovation,' *The Quarterly Journal of Economics*, pp. 395–410.
- Martin, A., 2009, 'A Model of Collateral, Investment, and Adverse Selection,' *Journal of Economic Theory*, 144(4), pp. 1572–1588.
- Ménière, Y., 2008, 'Patent Law and Complementary Innovations,' *European Economic Review*, 52(7), pp. 1125–1139.
- Meurer, M. and Strandburg, K., 2008, 'Patent Carrots and Sticks: An Economic Model of Nonobviousness,' *Lewis and Clark Law Review*, 12, pp. 549.
- Mitchell, M. and Zhang, Y., 2012, 'Shared Rights and Technological Progress,' *working paper, available at <http://ssrn.com/abstract=2116734>*.
- Mojibi, A., 2010, 'An Empirical Study of the Effect of KSR v. Teleflex on the Federal Circuit's Patent Validity Jurisprudence,' *Albany Law Journal of Science and Technology*, 20(3), pp. 101.
- Nock, J. and Gadde, S., 2010, 'Raising the Bar for Nonobviousness: An Empirical Study of Federal Circuit Case Law Following KRS,' *Federal Circuit Bar Journal*, 20, pp. 369.
- O'Donoghue, T., 1998, 'A Patentability Requirement for Sequential Innovation,' *The RAND Journal of Economics*, pp. 654–679.
- Rothschild, M. and Stiglitz, J., 1976, 'Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,' *The Quarterly Journal of Economics*, pp. 629–649.
- Schuett, F., 2013, 'Inventors and Impostors: An Analysis of Patent Examination with Self-Selection of Firms into R&D,' *Journal of Industrial Economics*, 61(3), pp. 660–699.
- Scotchmer, S., 1996, 'Protecting Early Innovators: Should Second-Generation Products be Patentable?,' *The Rand Journal of Economics*, pp. 322–331.
- Shapiro, C., 2000, 'Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard-Setting,' *Innovation Policy and the Economy*, 1, pp. 119–150.
- Takalo, T. and Toivanen, O., 2011, 'Entrepreneurship, Financiership and Selection,' *The Scandinavian Journal of Economics*, 114(2), pp. 601–628.
- Thomas, J., 2001, 'Collusion and Collective Action in the Patent System: A Proposal for Patent Bounties,' *University of Illinois Law Review*, 2001, pp. 305–353.
- Tykvová, T., 2007, 'What do Economists Tell Us about Venture Capital Contracts?,' *Journal of Economic Surveys*, 21(1), pp. 65–89.
- Van Dijk, T., 1996, 'Patent Height and Competition in Product Improvements,' *The Journal of Industrial Economics*, pp. 151–167.
- Witherspoon, J., 1980, *Nonobviousness—The Ultimate Condition of Patentability: Papers Compiled in Commemoration of the Silver Anniversary of 35 USC 103*. (Bureau of National Affairs, Bloomberg BNA, Arlington, Virginia, U.S.A.)

SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article at the publisher's web-site: joi_e_12031_sup_web_appendix.