

# Testing for Neglected Nonlinearity in the Conditional Quantile Using Neural Networks\*

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## Abstract

We propose a specification test for the conditional quantile by comparing a linear model to an alternative model represented by a neural network. The test is formulated as a testing problem with nuisance parameters under the alternative. We consider three test statistics, the *average*, the *exponential-average* and the *supremum Wald* statistic as functions of a Wald process and derive their asymptotic distribution. To simulate critical values, we propose a consistent bootstrap procedure. Further, we show the asymptotic optimality of the *average* and *exponential-average Wald* test against local alternatives in a correctly specified maximum likelihood setting. In an empirical application, we examine the nonlinearity of systemic risk in the framework of conditional value-at-risk (CoVaR).

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